Optimization of Reliability Testing and Multi-stage Production Quality Control Based on Binomial Sequential Testing

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Abstract. In modern manufacturing, enterprises are in urgent need of an optimized inspection solution that can guarantee product quality while reducing the sample size and cost for inspection, to address the challenges posed by high costs and strict reliability requirements. This study commences from multiple scenarios of finished product processing and component assembly procedures, based on the decision-making issues during the enterprise production process. A binomial sequential inspection model is established to obtain a suitable sampling inspection plan under binomial sequential inspection: When the confidence level is 95%, the maximum sample size is 6,355, and only 489 components must be inspected. If the defect rate significantly exceeds 10%, this batch of components will be rejected. When the confidence level is 90%, the maximum sample size is 5,293, and only 260 components must be inspected. If the defect rate does not exceed 10%, this batch of components will be accepted. Additionally, models such as the multi-stage production quality control optimization model based on dynamic programming and linear programming, and the multi-process decision optimization model have been established for systematic decision-making.

Keywords: Binomial Sequential Testing Model, Multi-stage production quality control optimization model, Dynamic programming, Linear programming.

1. Introduction

In modern manufacturing and high-tech industries, production quality control [1] and reliability testing [2] have become an important part of enterprise competitiveness. With the increase in market competition and product complexity, how to reduce testing and production costs while ensuring product quality has become one of the major challenges faced by enterprises. Especially in precision instruments [3,4], aerospace [5-7], medical equipment [8,9], and other fields, due to the high cost of parts and finished products and strict reliability requirements, traditional large sample detection methods are often impractical. In this case, enterprises need an optimization solution that can reduce the sample size, shorten the inspection time, and save costs while ensuring product quality.

The purpose of this study is to provide a systematic decision-making framework for enterprises to optimize their decision-making in production processes such as parts procurement, finished product assembly and quality inspection. Through the construction of mathematical models, the optimal sampling inspection scheme under different defective rates and confidence levels is determined, and effective strategies are formulated in the production and quality inspection stages to minimize the cost of enterprises and maximize the market reputation and profits of enterprises.

By optimizing sampling testing schemes and quality testing strategies in this study, enterprises can reduce unnecessary testing costs, improve product quality, reduce defective rates, and thus enhance market competitiveness. It can also provide a scientific decision-making tool for enterprises to help managers make optimal decisions quickly and improve management efficiency when facing different defective rates and confidence levels. By improving product quality, reducing the flow of defective products into the market, protecting the rights and interests of consumers, and enhancing the social responsibility and reputation of enterprises.

The innovation of this study lies in the combination of statistics and operations research methods to build a comprehensive sampling detection model suitable for different defective rates and confidence levels, so as to minimize the number of detections. In addition, from parts procurement, finished product assembly to quality testing, a multi-level quality testing strategy has been developed, considering multiple links such as disassembly and exchange, providing comprehensive decision support for enterprises. Finally, in the decision-making process, cost-benefit analysis is introduced to help enterprises weigh detection costs, dismantling costs, replacement losses, etc., to maximize the overall benefit.

This study first selects the appropriate sampling detection method according to the production mode of the enterprise and consults the literature. On the basis of consulting relevant literature, the binomial sequential test[10] and cost models of various cases are established. A multi-stage production quality control optimization model based on dynamic programming and linear programming is established, and then various parameters are introduced and combined with dynamic programming to make the optimal decision.

2. Development of Binomial Sequential Testing Model

Data in this study are derived from www.mcm.edu.cn. First of all, this study assumes that the production scenario of an enterprise is that an enterprise produces a best-selling electronic product, which needs to purchase two kinds of spare parts (spare parts 1 and spare parts 2) respectively, and assemble the two spare parts into finished products. In the assembly of the finished product, as long as one of the parts is unqualified, the finished product must be unqualified; If both parts are qualified, the finished product may not be qualified. For unqualified finished products, the enterprise can choose to scrap, or disassemble them, the disassembly process will not cause damage to the parts, but it needs to spend the disassembly cost. The supplier claims that the defective rate of a batch of parts (Part 1 or Part 2) will not exceed a certain nominal value. The enterprise intends to adopt a sampling testing method to decide whether to accept the parts purchased from the supplier, and the testing cost shall be borne by the enterprise itself. Assuming that the nominal value is 10%, this paper assumes the following two situations and gives corresponding results respectively: (1) If the defective rate of parts is determined to exceed the nominal value with 95% reliability, the parts will be rejected; (2) Under 90% reliability, it is determined that the defective rate of spare parts does not exceed the nominal value, and this batch of spare parts is received. Now this study needs to design a sampling inspection scheme for enterprises with as few inspection times as possible under this situation. Based on the above scenario, the basic process of hypothesis testing is made clear in Figure 1: first, the hypothesis is established, the significance level is determined, the prerequisite conditions are verified, the test statistics and rejection domain are determined, the test statistics are calculated according to the samples, and the judgment is made according to the test method. Finally, the actual conclusion is drawn.

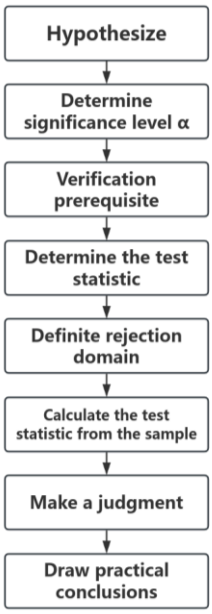

Figure 1. Basic flow chart for hypothesis testing

To better obtain the enterprise parts inspection scheme under this scenario, this paper establishes a binomial sequential inspection model. Regarding the problem of large samples in statistical inference, many classical statistics have been studied, and many useful conclusions have been given. However, in the field of reliability, such as precision instruments and expensive weapons, it is impossible to determine too large sample size in this study due to the need to consider the cost and time required for the test. Appropriate small-sample sampling schemes must be studied. For designing sampling detection schemes with as few detection times as possible, this paper mainly considers the following effects:

① Since the testing cost is borne by the enterprise itself, as few testing times as possible can help the enterprise save costs.

② Required parameters of the model: is the significance level, the calculation method is 1-reliability level, and different significance levels are used for different situations; β is the opposite of detection efficacy, its calculation method is 1- detection efficacy, and different detection efficacy is used for different situations; p' is the probability of success in the original hypothesis, that is, the nominal value given by the question (10%); p^* is the probability of success in the alternative hypothesis, which is generally chosen to be slightly greater than p'.

According to the basic process of hypothesis testing, it is necessary to first describe the null hypothesis and the alternative hypothesis, where the null hypothesis is that based on the given significance level, the defective rate of parts exceeds the nominal value, that is $p > p'$, The alternative assumption is that the defective rate of parts does not exceed the nominal value based on the given significance level, $p \le p'$.

On this basis, since the results of the hypothesis test follow the binomial distribution, sample size n can be calculated by the following formula:

$$n = \frac{\left(Z_{\alpha/2} + Z_{\beta}\right)^2 \cdot \left(p' \cdot (1 - p')\right)}{\left(p^* - p'\right)^2} \tag{1}$$

Among them: $Z_{\alpha/2}$ is the z-value corresponding to the significance level (for $\alpha = 0.05$, $Z_{\alpha/2} \approx 1.96$). is the z-value corresponding to the test efficacy(for efficacy= 0.80, $Z_{\beta} \approx 0.84$). p' is a nominal value (for example, 0.10). p^* is the defective rate in the alternative hypothesis.

For the first case of the enterprise's spare parts detection under this scenario, the reliability is 95%, the nominal value is 10%, and the defective rate of spare parts in the alternative hypothesis can be set to 15%, and the sample size n can be calculated.

For the second case of the enterprise's spare parts detection in this scenario, the reliability is 90%, the nominal value is 10%, and the defective rate of spare parts in the alternative hypothesis can be set to 15%, and the sample size n can be calculated.

It requires too much manpower, material resources, and financial resources to conduct all the tests on the obtained sample size, so the sequential test method is adopted in this paper to reduce the number of tests as much as possible. The sequential test adopts the test strategy of "try and see". Due to the use of the test process information, the test scheme can significantly reduce the average test time and average test sample size of the product sampling test compared with the fixed test sample size, thus greatly saving the test cost of the product sampling test.

According to the significance level and detection efficacy obtained above, two thresholds of the sequential test can be calculated, in which the upper threshold A is calculated by the following formula:

$$A = \frac{1 - \beta}{\alpha} \tag{2}$$

$$B = \frac{\beta}{1-\alpha} \tag{3}$$

After obtaining the two thresholds of the sequential test, it is necessary to construct a likelihood ratio function to make decisions for the thresholds. The likelihood ratio function is constructed by:

$$\Lambda = \left(\frac{p^*}{p'}\right)^X \left(\frac{1-p^*}{1-p'}\right)^{n-X} \tag{4}$$

At the same time, this paper also sets three sequential test parameters with an initial value of 0, which are expressed by S、F、T respectively. Where S represents the number of successes in the sequential test, F represents the number of failures in the sequential test, and T represents the total number of sequential tests, and the relationship between the three is $T = S + F$.

When the likelihood ratio is within the threshold value given above, the sequential test is successful, then $S+1$; When the likelihood ratio is not within the threshold value given above, the sequential test fails, then $F+1$; The final calculation T is determined by S and F at the end of the sequential test.

Then, $\hat{p}$ (the observed rate of defects) is calculated, which is the number of successful sequential tests/the total number of sequential tests. When $\hat{p} > p'$ (that is, the nominal value), the situation indicates that the inspection effect is significant, which means that the defective rate of parts is determined to exceed the nominal value with 95% confidence, and the batch of parts is rejected; On the contrary, when $\hat{p} \le p'$, the situation indicates that the null hypothesis cannot be rejected, which means that under 95% confidence, the defective rate of parts does not exceed the nominal value, and this batch of parts is accepted.

According to the above model establishment and analysis statistics, the final scheme of enterprise spare parts testing under this scenario is as follows:

(1) At a 95% confidence level, the maximum sample size is 6355, and only 489 parts need to be tested. If the defective rate significantly exceeds 10%, this batch of parts will be rejected.

(2) Under the 90% confidence level, the maximum sample size is 5293, only 260 parts need to be tested, and if the defective rate does not exceed 10%, this batch of parts will be received.

3. Multi-Stage Production Quality Control Optimization Model

Based on the previous section, to be closer to the production situation of the enterprise, the following assumptions are made in this study: (1) whether the spare parts (spare parts 1 and/or spare parts 2) are tested, if the spare parts are not tested, the spare parts will directly enter the assembly link; Otherwise, the detected unqualified parts will be discarded;

(2) Whether to test each assembled finished product, if not tested, the assembled finished product directly into the market; Otherwise, only qualified finished products enter the market;

(3) Whether to disassemble the detected unqualified products, if not disassemble, directly discard the unqualified products; Otherwise, repeat steps (1) and (2) for disassembled parts;

(4) For the unqualified products purchased by the user, the enterprise will unconditionally replace them, and generate certain exchange losses (such as logistics costs, corporate reputation, etc.). Repeat step (3) for returned nonconforming products. In addition, the product parameters in 6 different cases are set in Table 1. Now this research needs to make the optimal decision for each stage of the production process of the enterprise.

By selecting the load prediction results of 403 and 411 lines. We can see that the actual values of the lines basically match the predicted values, but there are also some errors, especially in the peak period of electricity consumption, as shown in Table.1 and Table.2.

Table 1. The parameters of parts 1 and 2 in the production of the enterprise

Case	Parts 1 Defective rate	Parts 1 Purchase price	Parts 1 Inspection cost	Parts 2 Defective rate	Parts 2 Purchase price	Parts 2 Inspection cost
1	10%	4	2	10%	18	3
2	20%	4	2	20%	18	3
3	10%	4	2	10%	18	3
4	20%	4	1	20%	18	1
5	10%	4	8	20%	18	1
6	5%	4	2	5%	18	3

Table 2. Parameters of finished products and unqualified products in production

Case	Finished product Defective rate	Finished product Purchase price	Finished product Inspection cost	Finished product Market price	Defective finished product Replacement loss	Defective finished product Dismantling cost
1	10%	6	3	56	6	5
2	20%	6	3	56	6	5
3	10%	6	3	56	30	5
4	20%	6	2	56	30	5
5	10%	6	2	56	10	5
6	5%	6	3	56	10	40

In this paper, the relationship between parts detection, parts, and cost is considered respectively, and the corresponding model parameters are defined.

① Consider the logic of whether each part is tested, whether the finished product is tested, and whether the final sale needs to be replaced and disassembled. Here are two examples:

(1) When parts 1, parts 2, and finished products are tested, the final finished products are qualified finished products, and can be sold directly without replacement and disassembly;

(2) When parts 1 is not tested and parts 2 is tested, and the finished product is also tested, part of the finished product obtained by the assembly is unqualified due to parts 1, and the finished product itself is unqualified, and the other part is unqualified due to parts 1 and parts 2, but the assembly link causes the finished product to be unqualified. At this time, it is necessary to consider whether to disassemble the finished product.

② Considering the relationship between the rate of defective parts and finished products and the inspection cost, and considering the proportion of disassembly cost and replacement loss of unqualified finished products in the market price, it is further concluded that when the ratio of defective product rate and inspection cost reaches a certain ratio, or the ratio of disassembly cost and replacement loss of unqualified finished products in the market price reaches a certain ratio, the enterprise can get the best decision plan.

③ Define model parameters: p_1 is the defective rate of part 1, p_2 is the defective rate of part 2, c_1 is the unit price of part 1, c_2 is the unit price of part 2, d_1 is the inspection cost of part 1, d_2 is the inspection cost of part 2, p_f is the defective rate of finished product, a is the assembly cost of finished product, d_f is the inspection cost of finished product, s is the market selling price of finished product, r is the replacement loss of unqualified finished product, t is Disassembly costs for nonconforming finished products.

To give the optimal decision scheme, this paper considers the value of total cost C under different circumstances and transforms the problem into a comparison of the size relationship of total cost C under various circumstances. The total cost C is obtained by adding the purchase price, inspection cost, disassembly cost, and replacement cost. According to the decision logic problems of each link in the model preparation, the multi-stage production quality control optimization model is established based on this, and the situation under this assumption is subdivided into six cases for processing.

Case 1: Do not test spare parts, do not test finished products, do not disassemble unqualified finished products, and calculate the total cost C .

$$C = c_1 \cdot p_1 + c_2 \cdot p_2 + p_f \cdot r \tag{5}$$

Case 2: Test parts 1, do not test parts 2, do not test finished products, do not disassemble.

$$C = d_1 + c_2 \cdot p_2 + a + p_f \cdot r \tag{6}$$

Case 3: Test parts 2, do not test parts 1, do not test finished products, do not disassemble.

$$C = d_2 + c_1 \cdot p_1 + a + p_f \cdot r \tag{7}$$

Case 4: Do not test parts 1 and 2, test the finished product, and do not disassemble.

$$C = c_1 \cdot p_1 + c_2 \cdot p_2 + a + d_f + p_f \cdot r \tag{8}$$

Case 5: Test parts 1 and 2, test the finished product and do not disassemble.

$$C = d_1 + d_2 + c_1 \cdot (1 - p_1) + c_2 \cdot (1 - p_2) + a + d_f + p_f \cdot r \tag{9}$$

Case 6: Test parts 1 and 2, test the finished product, and disassemble the unqualified finished product.

$$C_1 = d_1 + d_2 \tag{10}$$

$$C_2 = (p_1 \cdot (1 - p_2)) \cdot (d_1 + d_f) + (p_1 \cdot p_2) \cdot t + (p_2 \cdot (1 - p_1)) \cdot (d_2 + d_f) + p_1 \cdot p_2 \cdot t \tag{11}$$

$$C_3 = (p_2 \cdot (1 - p_1)) \cdot (d_2 + d_f) + (p_1 \cdot p_2) \cdot t + (p_1 \cdot (1 - p_2)) \cdot (d_1 + d_f) + p_1 \cdot p_2 \cdot t \tag{12}$$

$$C_4 = (p_1 \cdot p_2) \cdot t + ((p_1 \cdot (1 - p_2)) + (p_2 \cdot (1 - p_1))) \cdot (d_1 + d_2 + d_f) + p_1 \cdot p_2 \cdot r \tag{13}$$

$$C = C_1 + C_2 + C_3 + C_4 \tag{14}$$

In view of the different conditions of multi-stage production quality control optimization model parameters, the total cost C of different conditions can be obtained by inputting model parameters, and the case with the minimum total cost C is taken as the optimal decision scheme under this condition. Through dynamic programming, different model parameters are imported into the multi-stage production quality control optimization model, and the optimal decision in Table 3 is obtained in this study.

Table 3. Optimal decision based on multi-stage production quality control optimization model

Case	Optimal decision
1	Test parts 1 and 2, test finished products, and disassemble unqualified finished products
2	Test parts 1 and 2, test finished products, and disassemble unqualified finished products
3	Test parts 1 and 2, test finished products, and disassemble unqualified finished products
4	Test parts 1 and 2, test finished products, and disassemble unqualified finished products
5	Do not test spare parts 1, test spare parts 2, do not test finished products, do not disassemble
6	Test parts 1 and 2, test finished products, and disassemble unqualified finished products.

4. Conclusions

In this study, several scenarios are set up by the actual enterprise production, and the method of hypothesis testing is used to give the judgment results of different situations under different significance levels. Based on the null hypothesis and alternative hypothesis, the binomial distribution was used to determine the sample characteristics and the sequential test model was established by using the sequential test method to reduce the number of tests as much as possible, and finally, a suitable sampling test scheme was obtained: (1) Under 95% confidence level, the maximum sample size is 6355, only 489 parts need to be tested, if the defective rate significantly exceeds 10%, this batch of parts will be rejected; (2) Under the 90% confidence level, the maximum sample size is 5293, only 260 parts need to be tested, and if the defective rate does not exceed 10%, this batch of parts will be received. Given the need to make the optimal decision at each stage of the production process, this paper first analyzes the logical relationship between spare parts and finished products, including whether various spare parts are tested, whether the finished products are tested and disassembled, and whether the final sold finished products are exchanged and disassembled. Therefore, a multi-stage production quality control optimization model is established based on the multi-stage situation of dynamic programming and linear programming, and the model parameters are modified through the product parameters in different cases, the total cost of the enterprise under different circumstances is calculated, and the optimal decision scheme is determined with the lowest total cost.

This study reveals a research idea and framework applied to the field of quality control and management of enterprise production and proves the feasibility of the reliability test optimization and multi-stage production quality control model based on binomial sequential testing, which can reduce the sample size, shorten the testing time and save costs on the premise of ensuring product quality.

Future research directions will focus on how to further optimize sampling inspection schemes under different defective rates and confidence levels to minimize inspection times and costs. At the same time, considering the dynamic changes in the defective rate during the production process, develop real-time or regular quality control strategies to cope with possible quality fluctuations. Secondly, in-depth cost-benefit analysis, including inspection cost, dismantling cost, replacement cost, etc., to more accurately assess the impact of different decisions on the overall benefit of the enterprise, and establish a risk management and early warning system, so that quality problems before the emergence of timely detection and measures to reduce potential losses. Finally, how to cooperate with suppliers to jointly improve the quality of spare parts, so as to reduce inspection costs and defective rates.

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