# Research on Decisions in the Production Process Based on Dynamic Programming

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**Abstract.** Electronic products have become a necessity in daily life. This paper discusses the decision-making problems in the production process of an electronic product enterprise. Through various relevant analyses, establishing corresponding mathematical models, designing a reasonable sampling inspection scheme, determining the optimal decision in the production process, improving the qualification rate and quality of finished products, and enhancing the market competitiveness of the enterprise. Firstly, this paper judges whether to accept parts under two reliabilities through hypothesis testing, obtains the sample quantity by using the normal distribution approximation model, then conducts sampling inspection and judges whether to reject the null hypothesis, and finally uses the sequential probability ratio test to improve the model to make decisions with as few sampling times as possible.Secondly, this paper first considers the general situation of (m) processes and (n) types of spare parts in enterprise production, constructs a decision tree model to determine a more comprehensive decision-making scheme, then constructs a dynamic programming optimization model to analyze its costs and obtains an idealized decision-making scheme. It then analyzes the special situation of 2 processes and 8 types of spare parts to obtain a decision-making scheme, and finally uses a genetic algorithm to calculate the optimal decision for the special situation.

**Keywords**: Decisions in the production process, Normal distribution approximation, Sequential probability ratio test, Dynamic programming optimization.

## 1. Introduction

With the development of the economy and enterprises, users have increasingly higher requirements for product quality, and it is imperative to improve product quality to meet customer needs. Among them, the decisions made by enterprises in the production process are particularly important. By making optimal decisions, higher economic benefits can be obtained while ensuring that the product quality and performance meet consumer needs [1].

Existing decision analysis methods mainly include optimization theory, efficiency maximization, establishing special models, and making decision analyses based on specific backgrounds, etc., and have achieved good decision-making results. Zuo Naiqian [2] analyzed the optimal production decision of a manufacturing enterprise with limited production capacity under environmental regulations based on an optimization theory model; Chen Xinhuan [3] analyzed the influencing factors of input-output decisions in an open-pit coal mine from the dimension of efficiency maximization, etc.; Chen Wenchong [4] established a data-driven virtualization model of a discrete manufacturing system and analyzed the production process control decisions from order acquisition to on-time submission of the system; Shi Yu [5] analyzed the optimal decisions of manufacturers and retailers in a green supply chain with three government subsidy policies as the background.

In this paper, under a specific background, it first determines whether to accept spare parts through methods such as hypothesis testing and sequential probability ratio, then analyzes the decision-making scheme based on decision tree and dynamic programming models, and finally uses a genetic algorithm to obtain the optimal decision for special situations, in order to increase the enterprise's revenue and customer satisfaction.

# 2. Design a Sampling Inspection Scheme with as Few Inspection Times as Possible to Judge Whether to Accept Spare Parts

## 2.1. Background Introduction

The defective rate of spare parts will not exceed the nominal value of 10%. This paper first designs a sampling inspection scheme based on two situations, namely 95% reliability, rejecting if the defective rate exceeds the nominal value; 90% reliability, accepting if the defective rate does not exceed the nominal value, and making the inspection times as few as possible.

## 2.2. Using Hypothesis Testing to Judge Whether to Accept Spare Parts

Hypothesis testing is used to detect whether the defective rate (p) exceeds the nominal value ($P_0$) [6], that is, the null hypothesis ($H_0$): the defective rate does not exceed the nominal value, that is, ($p \leq 0.1$); the alternative hypothesis ($H_1$): the defective rate exceeds the nominal value, that is, ($p > 0.1$). When the reliability is 95%, if ($p > 0.1$), then reject ($H_0$); similarly, when the reliability is 90%, if ($p \leq 0.1$), then accept ($H_0$).

## 2.3. Normal Distribution Approximation of Binomial Distribution

Let the sample size be (n) and the number of defective items be (X), then (X) follows a binomial distribution with parameters (n) and (p) [7]. The number of this batch of spare parts is large enough, and both (np) and (n(1 - p)) are greater than 5, so this binomial distribution can be approximated by a normal distribution [8]. According to the central limit theorem, this binomial distribution can be approximated as a normal distribution with (np) as a mean and (np(1 - p)) as the variance.

## 2.4. Calculating the Sample Size (n)

The significance level of situation one is ($\alpha = 1 - 0.95 = 0.05$), and the critical value of normal distribution is ($Z_\alpha = 1.645$); the significance level of situation two is ($\beta = 1 - 0.90 = 0.10$), and the critical value of normal distribution is ($Z_\beta = 1.28$) To minimize the enterprise's inspection costs as much as possible and ensure the accuracy of the inspection results, the sample size needs to be optimized. The formula for calculating the sample size is as follows:

$$n = \frac{Z_\alpha \times P_0\left(1 - P_0\right)}{\left(P_0 - P_1\right)^2} \tag{1}$$

($P_1$) is the acceptable actual defective rate, ($P_0 - P_1$) represents the magnitude of error.

Assume that the ($P_1$) values in the two situations are 0.15 and 0.05 respectively. According to the formula (1), the sample values are 98 and 59 respectively.

## 2.5. Sampling Inspection and Judgment

Actual defective rate($\hat{P} = \frac{X}{n}$). Under 95% reliability, if the test statistic ($Z > 1.645$), reject ($H_0$), the defective rate exceeds the nominal value, reject; under 90% reliability, if ($Z \leq 1.28$), accept ($H_0$), the defective rate does not exceed the nominal value, accept.

## 2.6. Sequential Probability Ratio Test

To further reduce the number of inspections, this paper uses the SPRT to improve the model. This method can make real-time decisions during the inspection process according to the collected data and decide whether to continue sampling [9]. First, determine the two thresholds (A) and (B) of the sequential probability ratio test, then calculate the likelihood ratio( $L(X)$ ), and finally make a judgment:

(1) If ( $L(X) \geq A$ ), reject ($H_0$), that is, consider that the overall defective rate ($P > P_0$);

(2) If ( $L(X) \leq$ B), accept($H_0$), that is, consider that the overall defective rate ($P \leq P_0$);

(3) If ($B < L(X) < A$), continue sampling.

As the number of samplings increases, the likelihood ratio is continuously updated. Eventually, if it reaches or exceeds the threshold (A), the batch of spare parts is rejected; or if the likelihood ratio drops to or below the threshold (B), the batch of spare parts is accepted.

## 3. Decision-Making Problems in Actual Production

### 3.1. Background Introduction

In the actual production of enterprises, it is generally the case of (m) processes and (n) types of spare parts. Moreover, when enterprises assemble spare parts or semi-finished products, if any one is unqualified, the assembled product must be unqualified; if both are qualified, the assembled product may not necessarily be qualified. Assume that the defective rate and quantity of each type of spare part purchased by the enterprise are the same. Now, make decisions for the special situation of 2 processes and 8 types of spare parts to maximize the profit while minimizing the cost as much as possible. The explanation of processes and relevant data of spare parts, semi-finished products, and products are shown in Table.1.

**Table 1.** Process Explanation and Processing Information

| process | Assuming the process is Q (Q ≥ 1), if Q = 1, the spare parts are directly assembled into finished products; if Q > 1, the spare parts will be assembled into the first type of semi-finished product first, and then it will be assembled into the second type, and so on, until the last process. | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| spare part | defective rate and purchase price | | testing cost | semi- | defective rate | assembly cost | testing cost | dismantling cost |
| 1 | 10% | 2 | 1 | finished | 10% | 8 | 4 | 6 |
| 2 | 10% | 8 | 1 | product | 10% | 8 | 4 | 6 |
| 3 | 10% | 12 | 2 | 1 | 10% | 8 | 4 | 6 |
| 4 | 10% | 2 | 1 | 2 | 10% | 8 | 6 | 10 |
| 5 | 10% | 8 | 1 | 3 | | | | |
| 6 | 10% | 12 | 2 | product | market price | | exchange cost | |
| 7 | 10% | 8 | 1 | product | 200 | | 40 | |
| 8 | 10% | 12 | 2 | | | | | |

### 3.2. Establishing a Decision Tree Model

This paper first establishes a rational decision tree model for the general situation of (m) processes and (n) types of spare parts, and then applies it to the situation of 2 processes and 8 types of spare parts, that is, taking the relevant decision of spare part (i(i > 1)) as the root node, the relevant decision of semi-finished product (i) as the internal node, and the relevant decision of the finished product as the leaf node [10].

### 3.3. Constructing a Dynamic Programming Model

To optimize the decision, this paper constructs an expected total cost model based on dynamic programming to evaluate the corresponding costs [11], thereby calculating the total cost:

$$M_{\mathrm{sum}} = \sum_{i=1}^{n} \left( M_{\mathrm{spi}} + M_{\mathrm{ti}} + M'_{\mathrm{tij}} + M_{\mathrm{aij}} + M_{\mathrm{di}} \right) + M_{\mathrm{t}} + M_{\mathrm{d}} + M_{\mathrm{e}} + M_{1} \quad (2)$$

Where ($M_{\mathrm{spi}}$) represents the total purchase cost of the (i)-th type of spare part (i = 1 ~ n); ($M_{\mathrm{ti}}$ 、 $M'_{\mathrm{tij}}$ 、 $M_{\mathrm{t}}$) respectively represent the total testing cost of the (i)-th type of spare part, the (j)-th type of the (i) -th semi-finished product, and the finished product; ( $M_{aij}$ ) represents the total assembly cost of the (i)-th and (j)-th types of spare parts (i = 1 ~ n, j = 1 ~ n, j ≠ i); ( $M_{di}$ 、 $M_d$ ) respectively represent the total dismantling cost of the (i)-th semi-finished product and the finished product; ( $M_e$ 、 $M_l$ ) respectively represent the total exchange cost of the finished product and the total loss generated by the enterprise compensating qualified finished products to users.

### 3.3.1. Evolution Formula of Defective Rate

In the actual production process, due to the differences in the defective rate of each type of spare part and the defective rate of spare parts in each process, it is necessary to construct a formula that can deduce the final defective rate of the finished product, that is, the evolution formula of the defective rate:

$$P_n = 1 - \prod_{i=1}^{n} \left(1 - P_{1_i}\right) \tag{3}$$

where ( $P_{1_i}$ ) represents the defective rate of the ( i )-th type of spare part; ( $P_n$ ) represents the final defective rate of the finished product.

This formula indicates that in the production process, when the quality of each type of spare part meets the specified standard, the quality of the final finished product must meet the requirements. This formula accumulates the changes in the defective rate of each process and obtains the final defective rate of the finished product, and finally can realize the monitoring and control of the quality of spare parts in the production process.

### 3.3.2. Detection Cost of Spare Parts

In the production process, whether each type of spare part and semi-finished product is detected will affect the final cost.Now, calculate the costs of different decisions to estimate the overall cost:

$$M_t = \sum_{i=1}^{n} b_i M_{t_i} \tag{4}$$

Where ( $b_i$ ) and ( $M_{t_i}$ ) respectively represent the quantity and single testing cost of the (i)-th type of spare part (i = 1 ~ n).

### 3.3.3. Losses Due to Not Detecting Spare Parts

This paper divides semi-finished products into the (i)-th type and (j)-th type of semi-finished products (i = 1, 2, 3……, j = 1, 2, 3……) according to the order in which they appear in processing. And the defective rates of semi-finished products and finished products are the defective rates of products produced after assembling genuine spare parts, that is, the defective rates of semi-finished products and finished products are related to whether the spare parts and semi-finished products assembled into them are genuine.

Not testing spare parts: Unqualified spare parts are assembled into the first type of semi-finished product, with impacts: additional assembly costs, the defective rate of the first type of semi-finished product, and entering the decision of whether to test the first type of semi-finished product.

Decision 1: If tested, additional costs will be produced, and the tested unqualified products will enter the dismantling decision. If dismantled, additional costs will be produced, and the dismantled spare parts need to be tested, and the qualified spare parts need to be reassembled; the assembled first type of semi-finished product needs to be re-tested. (This is a cyclic process, and the number of cycles will change with the number of spare parts. For convenience, this paper assumes that the defective rate of the semi-finished product after dismantling and reassembly is very small and the cycling problem can be ignored. However, considering that there will still be corresponding cycling in actual production, this paper assumes that the cost generated by cycling is ($M_{\mathrm{uk}}$), considering that this paper's consideration situation is under ideal conditions so it is assumed that ($M_{\mathrm{uk}} \approx 0$).)

After being assembled into the second type of semi-finished product, the decision of whether to test is entered; if tested, then for unqualified products, the dismantling decision is entered. If dismantled, corresponding costs will be generated (the assembled semi-finished product has no defective products, so there is no additional dismantling cost). At this time, all the first type of semi-finished products dismantled are qualified because the defective rates of semi-finished products and finished products are the defective rates of products produced after assembling genuine spare parts. This decision has the same defective rate and will fall into a dead cycle, so this decision is excluded. If not dismantled, then the unqualified products are discarded and the rest continue to be assembled and enter the decision of whether to test the third type of semi-finished product.

It can be seen from this that after reassembling and testing the first type of semi-finished product after dismantling, if the decision is to test the second type of semi-finished product, for subsequent processes, the process of detecting, not dismantling, assembling, and testing will be repeated until the last process and finally flow into the market. Under the premise of ($M_{\mathrm{uk}} \approx 0$), this decision-making process will not generate any additional costs. In actual situations, if the number of spare parts is fixed and the number of processes is sufficient, this process can also be realized.

If not tested, then it is assembled into the fourth type of semi-finished product, with impacts: additional assembly costs, the defective rate of the fourth type of semi-finished product, and entering the decision of whether to test the fourth type of semi-finished product.

If tested, additional costs will be generated. If the decision is to detect and dismantle the (q - 1)-th type, reassemble and not detect the (q)-th type, assemble and detect the (q + 1)-th type, it is equivalent to repeating decision 1 several times until the cycle ends when. During the cycle, as the number of times increases, the defective rate of the semi-finished product will increase, and eventually, it may be the case that all subsequent processes are defective, so this decision is directly abandoned. In an actual situation, within one cycle, the defective rate increases by more than 10 percentage points, and the loss far exceeds the enterprise's expectation, so this decision can be directly abandoned.

If not tested, continue to assemble, which will also lead to an increasing defective rate of the semi-finished product. This decision is similar to the above decision situation, and in actual situations, the percentage increase in the defective rate is greater than that of the previous decision, so it can also be abandoned.

If not dismantled the first type of semi-finished product, then discard the unqualified products, assemble the qualified products, and then repeat the above decision.

In summary, the potential losses due to not detecting spare parts can be expressed as:

$$\begin{aligned} \frac{L_1'}{2} = {} & L_{\mathrm{a}1}' + X_{\mathrm{t_{i1}}}\left(\left(\sum_{\mathrm{i}=1}^{a} L_{\mathrm{t_{i1}}}'\right) + \left(\sum_{\mathrm{i}=1}^{a} X_{\mathrm{d_{i1}}} L_{\mathrm{d_{i1}}}'\right) + X_{\mathrm{d_{i1}}}\left(\sum\left(l_{\mathrm{t_{i1}}} + l_{\mathrm{a_{i1}}}\right) + M_{\mathrm{uk}}\right)\right) \\ & + \bar{X}_{\mathrm{t_{i1}}}\left[L_{\mathrm{a}2}' + X_{\mathrm{t_{i2}}}\left(\left(\sum_{\mathrm{i}=1}^{\mathrm{c}} L_{\mathrm{t_{i2}}}'\right) + \left(\sum_{\mathrm{i}=1}^{\mathrm{c}} X_{\mathrm{d}2} L_{\mathrm{d_{i2}}}'\right) + X_{\mathrm{d_{i2}}}\left(\sum\left(l_{\mathrm{t_{i2}}} + l_{\mathrm{a_{i2}}}\right) + M_{\mathrm{uk}}\right)\right)\right] \end{aligned} \tag{5}$$

$$\begin{cases} M_{\mathrm{un}} \approx 0 \qquad L_{\mathrm{a}1}' = \sum_{\mathrm{i}=1}^{\mathrm{n}} bM_{\mathrm{a_{i1}}} p_1 \qquad q = \sum_{\mathrm{y}=0}^{\mathrm{z}} C_{\mathrm{z}}^{\mathrm{y}} \left(1 - p_1\right)^{\mathrm{y}} p_1^{\mathrm{z\text{-}y}} p_{2_{\mathrm{i1}}}^{\mathrm{x}} \\ n_{\mathrm{i1}} = bq \qquad L_{\mathrm{t_{i1}}}' = M_{\mathrm{t_{i1}}}' n_{\mathrm{i1}} \qquad L_{\mathrm{d_{i1}}}' = M_{\mathrm{d_{i1}}}' n_{\mathrm{i1}} \qquad l_{\mathrm{t_{i1}}} = M_{\mathrm{t_{i1}}} n_{\mathrm{i1}} z \\ l_{\mathrm{a_{i1}}} = M_{\mathrm{a_{i1}}} \frac{m_{\mathrm{s_{i1}}}}{2} P_{\mathrm{cp_{i1}}} \qquad m_{\mathrm{s_{i1}}} = zn_{\mathrm{i1}} \qquad m_{\text{正}_{\mathrm{i1}}} = \sum_{\mathrm{n}=1}^{\mathrm{n}} nbP_{\mathrm{n}+1} \\ P_{\mathrm{cp_{i1}}} = \frac{m_{\mathrm{cp_{i1}}}}{m_{\mathrm{s_{i1}}}} \qquad L_{\mathrm{a}2}' = \sum_{\mathrm{i}=1}^{\mathrm{c}} M_{\mathrm{a_{i2}}} n_{\mathrm{i1}} \qquad n_{\mathrm{i}2} = \left(b - n_{\mathrm{i1}}\right) \sum_{\mathrm{e}=0}^{\mathrm{f}} \left(1 - p_{2_{\mathrm{i2}}}\right)^{\mathrm{f}} p_{2_{\mathrm{i2}}}^{\mathrm{e-f}} p_{2_{\mathrm{i3}}}^{\mathrm{x}} \\ L_{\mathrm{t_{i2}}}' = M_{\mathrm{t_{i2}}}' n_{\mathrm{i}2} \qquad L_{\mathrm{a}3}' = M_{\mathrm{a_{i3}}} n_{\mathrm{i}2} \end{cases} \tag{6}$$

Where ($L'_{a1}$) is the total assembly loss of unqualified spare parts; (b) is the quantity of a type of spare part; ($X_{t_{i1}}$) represents whether to test the first type of semi-finished product (if detected, $X_{t_{i1}}$ = 1; otherwise, it is 0) (i = 1 ~ a); ($M'_{t_{i1}}$) represents the cost of testing a first type of (i)-th semi-finished product; ($n_{i1}$) is the total amount of non-conforming products generated by not testing spare parts; (q) is the new defective rate of each type of first type of semi-finished product generated when unqualified spare parts enter the assembly process ((z) is the type of spare parts required for assembly, (y) is the possible quantity of genuine products in this type of semi-finished product, (z ≥ y), and when (z = y), (x = 1), when (z ≠ y), (x = 0)); ($p_{2_{i1}}$) is the fixed defective rate of the first type of (i)-th semi-finished product assembled from genuine spare parts; $\sum\left(l_{t_{i1}}+l_{a_{i1}}\right)$ represents that due to the uncertain combination of spare parts assembled into the first type of semi-finished product, the number of a certain type of spare part dismantled cannot be determined temporarily; ($m_{s_{i1}}$) is the total number of spare parts dismantled from the first type of (i)-th unqualified semi-finished product; ($P_{cp_{i1}}$) is the genuine rate of spare parts after dismantling the first type of (i)-th unqualified semi-finished product; ($\overline{X}_{t_{i1}}$ and $X_{t_{i1}}$) are mutually exclusive.

Due to the uncertainty of the number of processes, the risk of whether each type of semi-finished product is detected cannot be evaluated for the time being. For the special situation considered in this paper, considering that the dismantling and reassembly process of semi-finished products involves a series of decision-making difficulties as described above, for the convenience of calculation, this paper does not consider the dismantling decision of semi-finished products.

### 3.4. Calculating the Optimal Decision by Genetic Algorithm

For the special situation, this paper uses a genetic algorithm to solve it. The genetic algorithm can find a relatively good solution within a relatively small number of evolutionary generations. When dealing with multi-data problems, it does not need to conduct an exhaustive search to obtain satisfactory results, improving the efficiency of the algorithm [12]. Therefore, the optimal decision can be obtained by the genetic algorithm: do not test spare part 6, test the others; do not test semi-finished product 3, test the others; do not test the finished product, do not dismantle the unqualified finished product; the total cost is 132.58. The other data related to the genetic algorithm are shown in Figure 1.

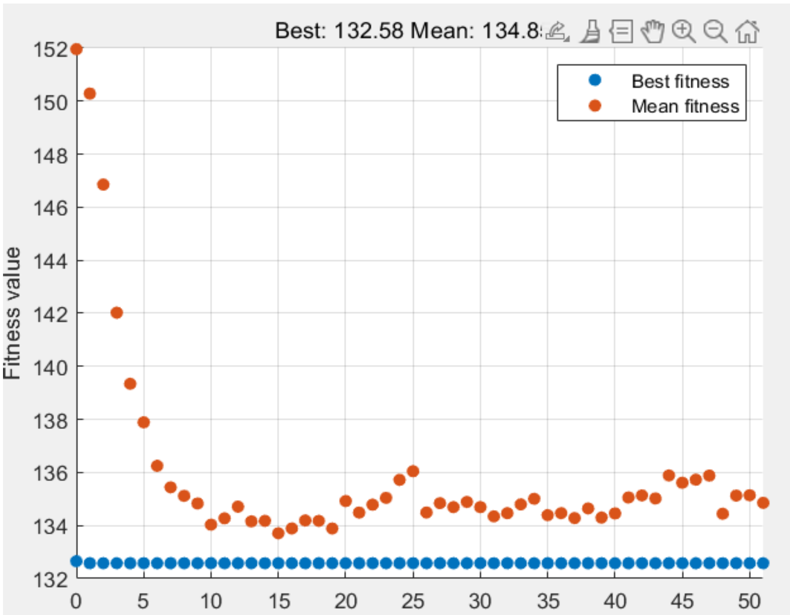


**Figure 1.** All situation data

## 4. Conclusions

Firstly, this paper uses hypothesis testing to establish the entire theoretical framework and uses the statistical method of normal distribution approximation of binomial distribution to ensure the rigor of the entire model and the reliability of the final sampling scheme. It also uses the SPRT to make the model more perfect, thus significantly reducing the number of tests and meeting the enterprise's needs.

Secondly, this paper obtains the decision-making situation in production by establishing a decision tree model, then establishes an expected total cost model based on dynamic programming and analyzes the costs and benefits brought by each decision in an ideal situation, and finally applies it to a special situation and obtains the optimal decision by a genetic algorithm.

The future research direction aims to consider more practical situations rather than ideal ones to make decisions in the production process of enterprises.

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