Enterprise production quality optimization model based on Markov chain

Huiling Yang [1], Aojie Lu [1], Aoyu Wang [2], Decheng Zhang [1, *]

[1] Public Basic Colloege, Bengbu Medical University, Bengbu, China, 233030

[2] College of Preventive Medicine, Bengbu Medical University, Bengbu, China, 233030

* Corresponding Author Email: zdc@bbmc.edu.cn

Abstract. In the 21st century, rapid technological advancements have presented various challenges in product quality management for enterprises. A comprehensive quality management system supports the entire product lifecycle, while effective production decision-making enhances competitiveness. Each step in the production decision process impacts profitability. Markov chains can be utilized to identify and define the states and behaviors of products throughout their journey from production to sale. This approach aids in determining optimal regression decision strategies for different components and processes. Such optimization improves market structure, reduces management costs, and ultimately boosts profits. By addressing these elements systematically, companies can navigate the complexities of modern manufacturing and maintain a competitive edge in the marketplace. Integrate advanced analytics with Markov chain models to enable predictive maintenance and quality prediction. This proactive approach can help companies anticipate problems before they escalate, ensuring improved product reliability and customer satisfaction in an increasingly competitive environment.

Keywords: Production Quality Management, Markov Chain, Decision Optimization, Profitability.

1. Introduction

With the transformation and upgrading of China's manufacturing industry and the increasing material and cultural needs of the people, consumers pay more and more attention to the quality of various products. Most of the products produced by small and medium-sized manufacturing industries are necessities in people's lives. Too many problems in the same batch of products will affect the user experience and reduce the integrity of enterprises. Building a sound quality management system is the key for small and medium-sized manufacturing enterprises to ensure product quality. In the operation of enterprises, quality management can ensure that products or services meet industry standards and customer expectations and maintain product quality consistency and compliance through inspection processes [1-4]. Taking benefits as the standard, efficient quality management decisions are proposed to improve the quality and efficiency of product production and promote the development of small and medium-sized manufacturing industries [5-6].

Nowadays, with the deepening of algorithms, the application of major artificial intelligence algorithms has improved the accuracy of quality detection [7]. Reduce processing costs, transportation costs, etc., and make clear planning for decision-making problems in processing spare parts products [8].

The Markov chain-based model for optimizing enterprise production quality is built upon production profits. It defines different product states and behaviors of products and establishes a transition reward matrix based on actual conditions [9-10]. The aim is to choose the lowest-cost decision-making method. This model can be used to evaluate the economic benefits of different product states and production behaviors. Markov chain model helps enterprises to develop optimal production strategy by modeling different states in the production process (products qualified, defective, and unqualified, etc.).

2. Markov chain model building

2.1. Regression decision model

We need to carry out cost planning from the inspection and dismantling of different batches of parts, finished products, and nonconforming products to pursue the best decision to maximize economic benefits. For each case, according to the indicators of different parts, the cost problem can be solved by constructing linear programming equations. In the disassembly judgment of unqualified finished products, the regression decision tree is adopted to make a judgment. Considering the cost problem of unqualified products, the cost probability analysis is made for disassembly or disassembly. The defective product is discarded directly, and if there is profit after dismantling, the defective product is dismantled and re-entered into the decision tree, and the maximum economic benefit is obtained through linear programming. If the user is regarded as an inspection, the product must have an inspection, and only when the product is iterated into a qualified product can there be profit. The fewer iterations of a product, the greater the profit will be. When the loss of product iteration exceeds the income of the product, the iteration of the product will be terminated, and this will be the loss benefit of the product under each decision.

The flow of the model in this section is shown in Figure 1.

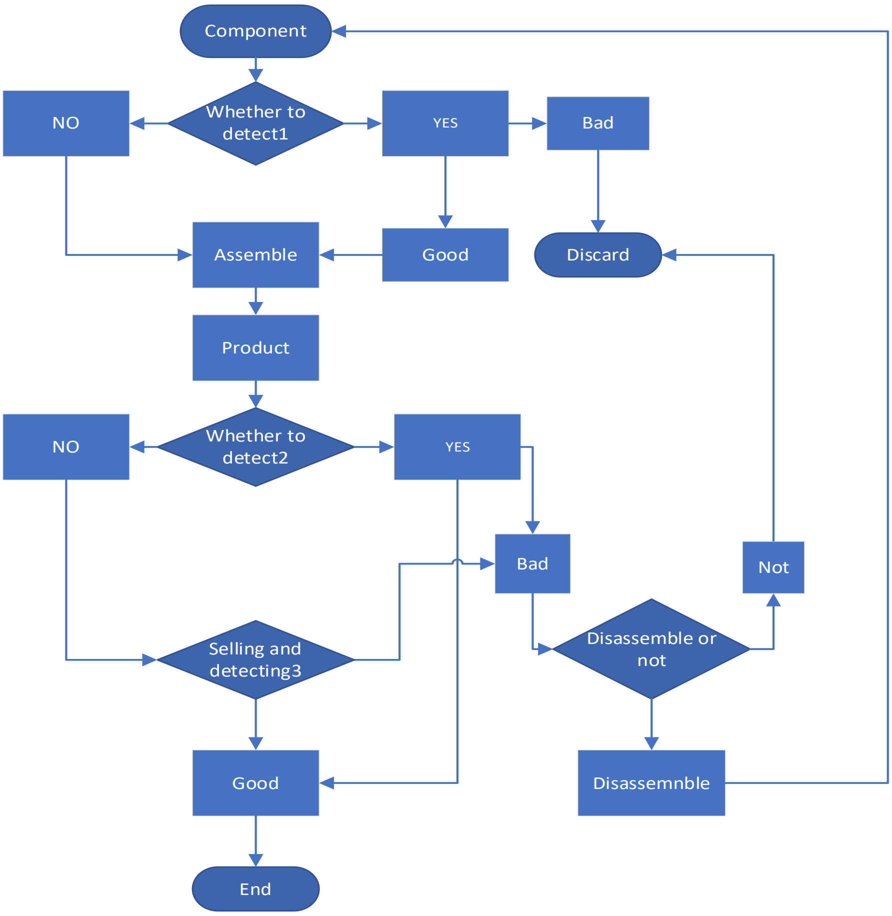

Figure 1. Parts processing flow.

$$P = R - C \tag{1}$$

Where P is the total profit of a product, and R is the unit price of a product. C is the total cumulative cost.

Here R is a fixed value, and this scheme is discarded when P is negative. When making decisions here, optimize the costs of parts using probability analysis. The unqualified products under different decisions can be divided into two categories, one is the unqualified products obtained by finished product inspection before sale, and the other is the unqualified products returned after sale. Each category of unqualified products has different disassembly ideas. Here, we construct a decision process to make the decision judgment.

$$C = \sum_{i=1}^{n} (P_i * Q_i * P_{ri}) \quad (2)$$

Where $P_i$the unit price of its type I parts/products, Q_i is the quantity of type I products, P_{ri} is the probability of non-conformity of type C products.

Through the calculation of the formula, with the probability of each part and the probability of the finished product, we can determine the optimal decision scheme by using profit as the criterion in the iterative calculation of product disassembly.

2.2. The establishment of Markov chains

In the actual problem, there are more than two parts that need to consider the cost problem. To quantify the actual problem, we need to consider the optimization of the production quality of m processes and n kinds of parts. Here, we take 2 processes and 8 parts as an example to establish a Markov chain model to find the best decision choice (and whether to detect or disassemble).

To implement value iteration, we need to initialize some variables: The value function of each state is initialized and then updated with the Bellman equation, and the value function is updated repeatedly until the optimal decision is obtained by running the model. Here, the entropy weight-Maldives decision model is used to deal with the decision problem of randomness and time series characteristics, and the semi-finished product, the detection and disassembly of the finished product are analyzed as states or actions. To find the optimal solution strategy.

The Markov chain is shown in Figure 2.

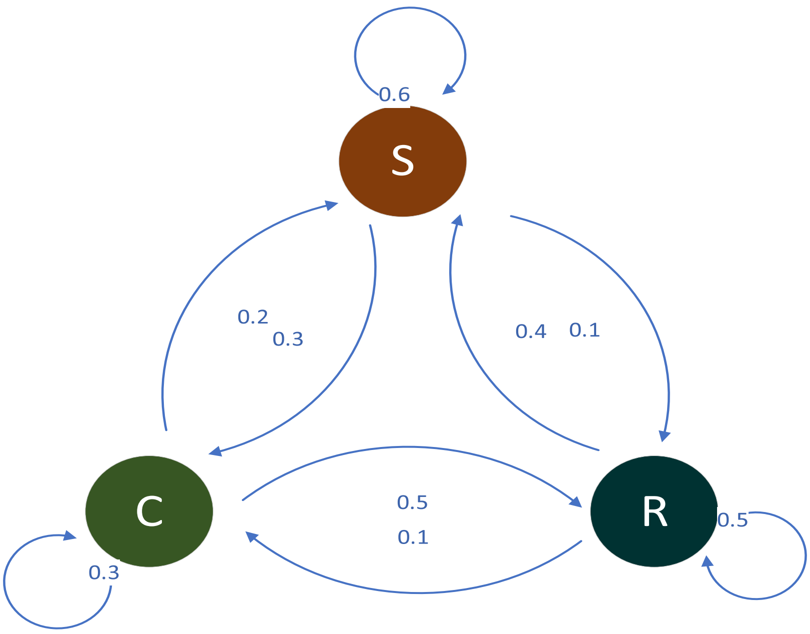

Figure 2. Markov chain example.

Define the probability transition matrix as a measure of the next decision.

$$P_{ij} = Pr(X_{n+1} = j | X_n = i) \quad (3)$$

Where P_{ij} is the probability of moving from state i to state j, and Pr represents the probability function. The sum of each row of the matrix P is 1, indicating that in any given state, the total probability of all possible next states is 1.

Define a reward matrix R, where each element R(s, a) represents a reward for taking an action an in states's.

Calculate the value function, value function: V(s) represents the expected return of adopting strategy π in states's. It can be calculated by the following Bellman equation:

$$V(s) = \sum_{a\in A} \pi(a|s) \sum_{s'\epsilon S} P(s'|s,a)[R(s,a) + \gamma V(s')] \tag{4}$$

Where A is the set of available actions. Gamma is a discount factor, usually between 0 and 1, used to discount future rewards.

In value iteration, the updated formula is:

$$V(s) \leftarrow \max \sum_{s'} P(s'|s,a)[R(s,a) + \gamma V(s')] \tag{5}$$

Finally, through the above process, the optimal strategy can be foundπ, and the corresponding optimal value function V, which will help optimize the production decision of the enterprise.

Start by defining states and actions. The Markov chain used by the model in this section is shown in Figure 3.

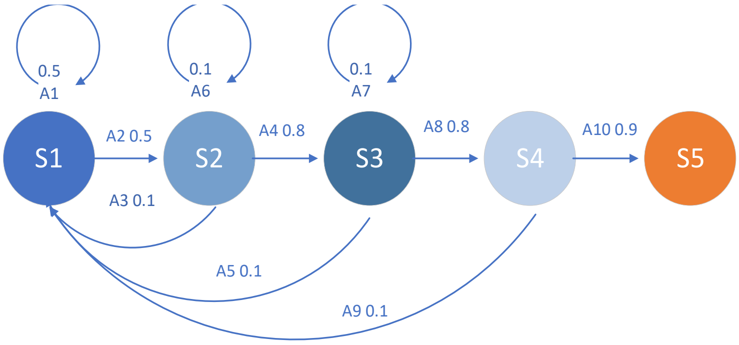

Figure 3. Example Markov chain.

In this Markov chain model, each state represents a crucial phase in the production lifecycle. S1 (raw material) marks the starting point, where raw inputs are gathered. S2 (semi-finished product) occurs after initial processing, while S3 (finished product) signifies the completion of manufacturing. S4 (sold finished product) tracks items that have been sold to customers. Finally, S5 (revenue finished by-product) captures the financial aspect, representing income generated from sales.

The actions (A1 through A10) facilitate transitions between these states. For example, A1 involves self-inspecting parts to ensure quality, whereas A8 is focused on selling the finished product. The probability transition matrix quantifies the likelihood of moving from one state to another based on current actions, providing insights into production dynamics. Meanwhile, the reward matrix assigns values to immediate rewards derived from specific actions taken within certain states, guiding decision-making toward maximizing overall profitability and efficiency in the production process. This structured approach enables enterprises to strategically navigate the complexities of modern manufacturing environments.

The sample data used in the model in this section comes from Question B of mathematical modeling in 2024, as shown in Table 1.

Table 1. Sample data.

State	Action	Transition probability	P
S1	A1	S1-S1	0.5
S2	A2	S1-S2	0.5
S3	A3	S2-S1	0.1
S4	A4	S2-S3	0.8
S5	A5	S3-S1	0.1
	A6	S2-S2	0.1
	A7	S3-S3	0.1
	A8	S3-S4	0.8
	A9	S4-S1	0.1
	A10	S4-S5	0.9

3. Results

3.1. The establishment of simulation model 1

In this section, we used a regression decision model to solve a simple production decision problem, the data in Table 2 can be obtained, where 1 represents the execution behavior and 0 represents the non-execution. The results are shown in Table. 2.

Table 2. The profit size under different decision circumstances.

Part 1/2 /Product	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
000	44	44	42.5	20	44	40
001	47	47	47	48	48	47
100	47.06	46.24	44.4	38.4	40.9	47.12
110	45.5	45.5	45.5	48.4	42	45.5
010	47.06	47	42.5	38.4	40	47.12
001	44	46.24	44	47	47	44
101	44.07	46.24	44.7	18.4	48	44.85
111	42.5	42.5	42.5	46.4	40	42.5

The optimization decision of case 1 is 100 or 010, the optimization decision of case 2 is 001 or 010, the optimization decision of case 3 is 001, the optimization decision of case 4 is 110, the optimization decision of case 5 is 101 and the optimization decision of case 6 are 100 or 010.

The optimal profit decision chain is obtained by comparing the profit corresponding to the decision under different circumstances. Corresponding to Table 2, it is clear that different cases tend to 001, of course, the optimal decision for parts and products with different defective rates is different. Through this model, the production decision problem of simple processed products can be solved, but for multiple processes, a variety of parts, different decisions corresponding to different parts and products have a variety of cases to calculate and consider, here we can use more advanced algorithms to continue to optimize the model.

3.2. The establishment of simulation model 2

The optimal production decision path can be obtained by optimizing the Markov chain model, and the example results are shown in Figure 4.

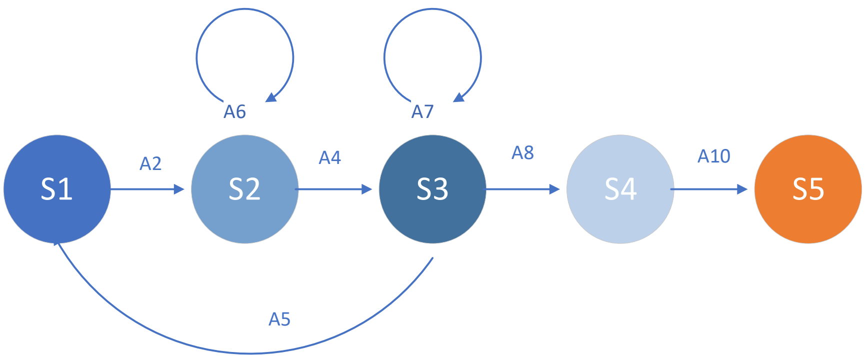

Figure 4. Decision result.

It can be seen from the results that the parts are not tested and self-tested after processing into semi-finished products, and the semi-finished products are directly discarded after processing into finished products. If the tested defective products are disassembled and put into production, if the sold products are detected as defective by users, they are directly discarded after being returned.

The above results are optimized decision models for 2 processes and 8 parts.

Model 2 has made more flexible improvements to Model 1. According to different processes and different parts, it can achieve multi-input and multi-output and realize data iteration in a single case through self-learning.

4. Conclusions

This paper outlines a comprehensive approach to optimizing product quality management for small and medium-sized manufacturing enterprises in China, focusing on the transition to an advanced quality management system due to increasing consumer demand for quality, and introduces a Markov chain-based model to optimize production quality, which defines various states and behaviors of products. The model contains a transition reward matrix that helps to make low-cost decisions and maximize production profits, and the model emphasizes cost planning based on inspection and disassembly of various batches of parts and products. By using linear programming and regression decision trees, companies can evaluate the cost-effectiveness of reworking defective products versus discarding defective products. The goal is to maximize economic benefits while minimizing product iteration cycles.

This comprehensive framework leverages advanced algorithms and decision processes to enhance the quality management systems of small and medium-sized manufacturing enterprises. By focusing on cost optimization, quality control, and data-driven decision-making, manufacturers can effectively respond to consumer demand while maximizing profitability. Further advances in algorithms can continue to improve these models for more complex production environments.

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