Research on Multi stage Production Process Optimization Based on Bayesian Inference Model

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Abstract. Supply chain optimization not only affects the production efficiency and product quality of enterprises, but also directly impacts their cost control and profitability. This article focuses on the spare parts provided by suppliers, the multi-stage production process of enterprises, and quality control during the production process. A Markov decision process was used to construct an optimization decision model aimed at improving production efficiency and reducing defect rates through sampling testing and cost optimization. This provides decision-making support and a basis for optimizing the production process for enterprises.This article makes a decision on whether a company should purchase a batch of spare parts with a defect rate not exceeding a certain nominal value while minimizing the number of inspections. Describing the sampling process with a binomial distribution, establishing a confidence level based sampling detection model to estimate the defect rate, using a one-sided inspection method to determine whether the defect rate of spare parts exceeds the nominal value by minimizing the number of inspections, and making the optimal decision for the enterprise. Based on this, the acceptable defect rate for enterprises is limited to 0.05~0.2. Using statistical methods, when the reliability is 95%, the defect rate is 0.199, corresponding to the minimum sample size that needs to be tested, which is 104; When the reliability is 90% and the defect rate is 0.063, the corresponding minimum sample size to be tested is 335. At the same time, the sequential probability ratio test was used in the article to optimize the model. By selecting a small number of samples and making a decision on whether to continue sampling based on the results, the number of experiments was saved under the same reliability.

Keywords: Supply Chain Optimization, Sampling Testing, Sequential Probability Ratio Testing.

1. Introduction

In modern manufacturing supply chain management and multi-stage production processes [1], quality control is an important link to enhance enterprise production efficiency and control costs. Currently, many enterprises still rely on traditional statistical methods in quality inspection, such as sampling inspection based on binomial distribution [2]. Although these methods have certain effects on detecting defective rates, they often have problems of excessive inspection costs or insufficient decision optimization when facing complex multi-stage production processes.

In recent years, Bayesian inference models and Markov decision processes have become increasingly important tools for decision optimization in multi-stage production processes. By combining sampling inspection and dynamic optimization, enterprises can make more efficient decisions in uncertain quality control environments, reduce the rate of defective products, and reduce production costs [3].

This paper studies the decision optimization of multi-stage production processes based on Bayesian inference models and proposes a production cost optimization scheme using Markov decision processes [4]. By constructing a decision model and combining sampling inspection and sequential probability ratio test, this paper effectively reduces the number of inspections and optimizes quality management in the production process through dynamic Bayesian networks, greatly improving the efficiency and accuracy of decision-making. The Bayesian inference model and Markov decision process used in this article have become important tools for decision optimization in multi-stage production processes. By combining sampled products with dynamic optimization, enterprises can make more effective decisions and reduce the rate of defective products in uncertain quality control environments.

2. Research on the defect rate of parts based on binomial distribution

2.1. Binomial distribution model

This article uses a hypothesis testing model to solve the problem, using one-sided testing to determine whether the defect rate of spare parts is greater than the nominal value ($p_0 = 10\%$). Firstly, make null hypothesis H_0 and alternative hypothesis H_1 [5-6].

(1) Conditions for rejection and acceptance

If the defect rate of the detected spare parts exceeds the nominal value with a 95% confidence, that is, in the case of equation (1), reject this batch of spare parts.

$$H_0 : p \leq 0.1 \tag{1}$$

$$H_1 : p > 0.1 \tag{2}$$

If the defect rate of the detected spare parts does not exceed the nominal value with a reliability of 90%, that is, in the case of equation (2), accept this batch of spare parts.

$$H_0 : p \geq 0.1 \tag{3}$$

$$H_1 : p < 0.1 \tag{4}$$

Since the distribution of defect rates follows a binomial distribution model, a binomial distribution can be used to describe the sampling process and calculate the sample size to be extracted.

(2) Selection of statistical measures

Using the proportion of non-conforming products in the sample as the detection value for the defect rate, assuming the sample size is n and the number of defective products obtained from sampling is x, the sample defect rate is:

$$\hat{p} = \frac{x}{n} \tag{5}$$

X conforms to the binomial distribution of sample size and defect rate p:

$$x \sim B(n, p_0) = B(n, 0.1) \tag{6}$$

When the defect rate is the nominal value, the corresponding probability of event occurrence is calculated from the sample size of 0, as shown in equation (3)

$$p(x = k) = C_n^k p_0^k (1 - p_0)^{n-k} \tag{7}$$

2.2. Does the normal approximate binomial distribution test reject the null hypothesis

According to the Central Limit Theorem, when the sample size is large enough, it is assumed that the sample defect rate $\hat{p}$ follows a normal distribution, with a standard deviation of [7]:

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} \tag{8}$$

According to different confidence levels, the cumulative probability of normal distribution can be used to calculate the corresponding sampling size n [8].

Let α be the level of significance required by the enterprise, and β be the probability of rejection of incorrect decisions that the enterprise hopes for. If the test has a low statistical test power β , repeated experiments are prone to make type 2 errors, which may mistakenly retain H_0 and hinder meta-analysis. After conducting a test power analysis, the test effectiveness is taken $1-\beta=0.92$, i.e. $\beta=0.08$.

Then there are:

$$n=\frac{(Z_{\alpha/2}+Z_{\beta})^2\cdot p_0(1-p_0)}{(p_1-p_0)^2} \tag{9}$$

The calculation formula for the normality test statistic Z is as follows:

$$Z=\frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \tag{10}$$

Among them, $\hat{p}$ is the defect rate in the sample, and p_0 is the nominal value. Z_{β} Represents the critical value of normal distribution corresponding to the test force, and $Z_{\alpha/2}$ represents the critical value of normal distribution corresponding to the significance level. For a 95% confidence level, $Z_{\alpha/2}\approx 1.96$ for a 90% confidence level, $Z_{\alpha/2}\approx 1.645$, p_1 represent the defect rate that the enterprise wishes to detect.

(1) Calculate whether to accept this batch of parts using confidence intervals

For different sample defect rates, the upper and lower limits of their confidence intervals can be calculated using equation (11).

$$\begin{cases}\hat{p}_{\text{upper}}=\hat{p}+Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\\ \hat{p}_{\text{below}}=\hat{p}-Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\end{cases} \tag{11}$$

If the upper limit estimated by the confidence interval is less than the nominal value, i.e. $\hat{p}_{\text{upper}}\le p_0$, the spare parts will be accepted, or if the lower limit estimated by the confidence interval is greater than the nominal value, i.e. $\hat{p}_{\text{lower}}>p_0$, the spare parts will be rejected.

2.3. Model solving

By using MATLAB software to program and solve the above model, the sampling and testing results of the enterprise can be obtained as follows.

2.3.1. Reject spare parts with a defect rate exceeding 10% with 95% confidence

When the reliability is 95%, the significance level $\alpha=1-0.95=0.05$ is taken. Since the highest defect rate of spare parts and finished products encountered by the enterprise in production is 20%, it is assumed that the highest acceptable defect rate for the enterprise is 20%. Using MATLAB software programming with a step size of 0.001, the actual defect rate range is set, and a random number seed is generated as the actual defect rate sampled. The corresponding minimum number of detection samples and sample defect rate are calculated using equation (7), and a visual image is generated, as shown in Figure 1.

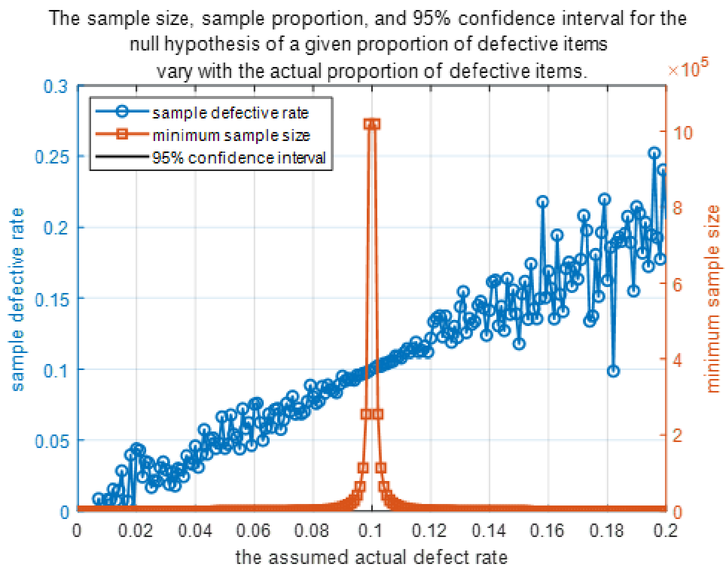

Figure 1. Changes in sample defect rate, minimum sample size, and 95% confidence interval with assumed actual defect rate.

In this case, it is only necessary to focus on the situation where the actual defect rate exceeds 10%. As shown in Figure 1, the larger the assumed actual defect rate, the smaller the sample size required for detection. At this time, there is a significant error between the actual defect rates generated based on the random number of seeds and the assumed defect rate. For example, when the assumed actual defect rate is 0.19, the randomly obtained defect rate is 0.26.

The maximum reasonable error between the assumed defect rate and the generated defect rate is further limited to 0.02, resulting in the variation shown in Figure 2.

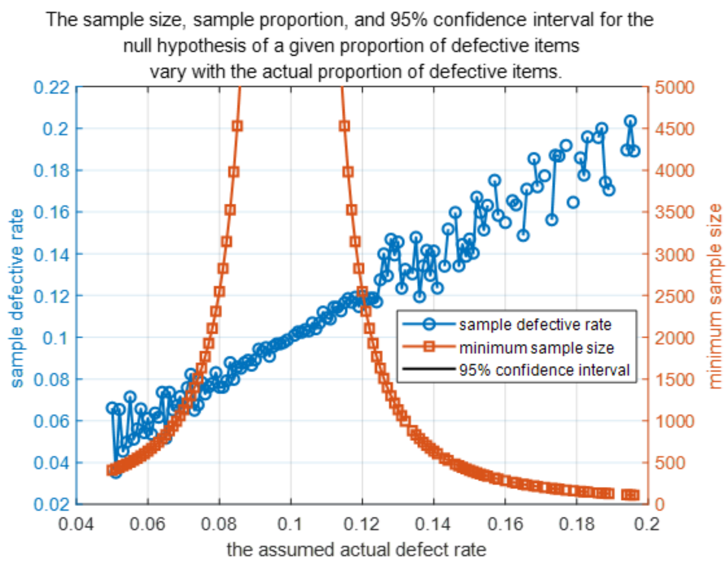

Figure 2. The variation of minimum sample size and 95% confidence interval with the assumed actual defect rate.

At this point, discontinuous points appear in the image. When the error range is within 0.1~0.2, assuming a defect rate of 0.199, the minimum number of samples to be detected is 104.

2.3.2. Receive spare parts with a defect rate of no more than 10% under 90% reliability

When the reliability is 90%, the significance level $\alpha = 1 - 0.90 = 0.10$ is taken. Since the lowest defect rate of spare parts and finished products encountered by the enterprise in production is 5%, it is assumed that the lowest defect rate encountered by the enterprise is 5%. With a step size of 0.001, the relationship between the number of samples to be tested and the assumed defect rate is plotted using MATLAB software as shown in Figure 3.

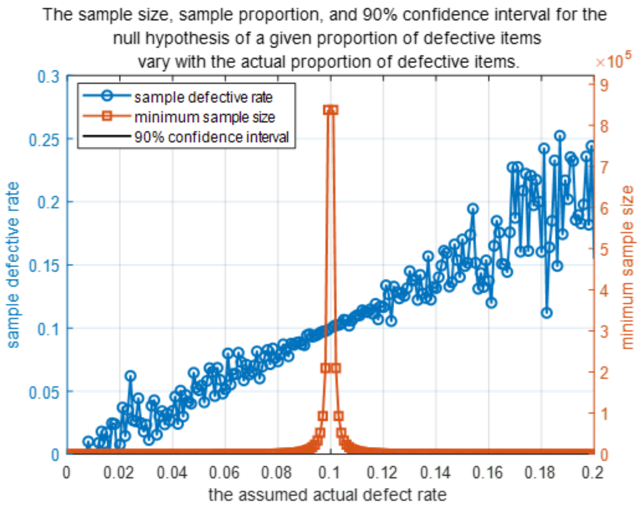

Figure 3. Changes in sample defect rate, minimum sample size, and 90% confidence interval with assumed actual defect rate.

In this case, it is only necessary to focus on the situation where the actual defect rate is less than 10%. As shown in Figure 3, the smaller the assumed actual defect rate, the smaller the sample size required for detection. At this time, there is a significant error between the actual defect rates generated based on the random number of seeds and the assumed defect rate. Similarly, the maximum reasonable error between the assumed defect rate and the generated defect rate is 0.02, resulting in the relationship shown in Figure 4.

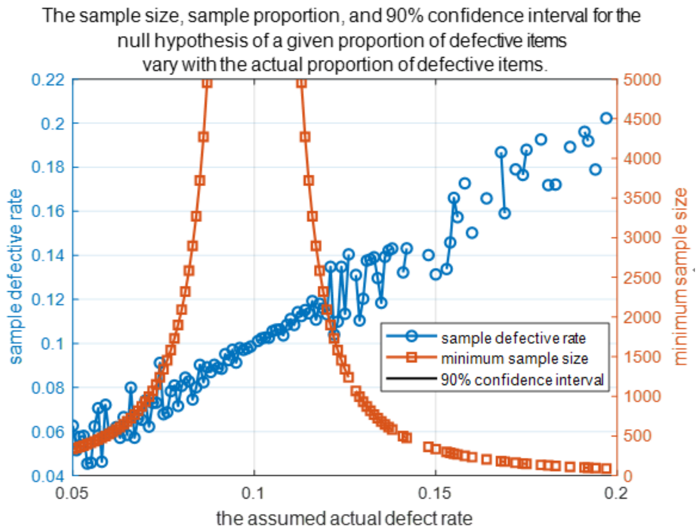

Figure 4. The variation of minimum sample size and 90% confidence interval with the assumed actual defect rate.

At this point, there are discontinuous points in the image. When the sample defect rate is within the range of 0.05~0.2, assuming a defect rate of 0.5, the resulting sample defect rate is 0.0626866, which corresponds to the minimum number of samples that need to be detected, which is 335.

2.4. Model optimization: Sequential probability ratio test

Detecting x yields $x_1, x_2, \cdots$. Let them be a set of independent and identically distributed random variable sequences, where $x_1, x_2, \cdots, x_n$ represent n random variables. For such a population sample value, SPRT provides two hypotheses for the two states of product production (qualified or unqualified):

Hypothesis H_0: When the product is qualified, the model's predicted deviation distribution follows a normal distribution with a mean of μ_0 and a variance of σ_0^2; Alternative hypothesis H_1:

When the product is unqualified, the model's predicted deviation distribution follows a normal distribution with a mean of μ_1 and a variance of σ_1^2 [9-10].

Their joint probability density can be defined as:

$$f_1(x) = p(x_1, \cdots, x_n \mid H_j) = \prod_{i=1}^{n} f(x_i / \theta_j) \tag{12}$$

Among them, j=0, 1, the likelihood ratio of the test is calculated as follows:

$$\lambda_n(x) = \lambda_n(x_1, \cdots x_n) = \frac{p(x_1, \cdots x_n \mid H_1)}{p(x_1, \cdots x_n \mid H_0)} = \frac{\prod_{i=1}^{n} f(x_i / \theta_1)}{\prod_{i=1}^{n} f(x_i / \theta_0)} \tag{13}$$

In the formula, $f(x/\theta)$ represents the conditional probability distribution and is a distribution parameter.

In hypothesis testing, for the random variable $x_1, x_2, \cdots,\ x_n$, given its probability density and proposing two hypotheses, selecting an appropriate constant C as the threshold, when $\lambda_n > C$, accept null hypothesis H_0; when $\lambda_n < C$, reject null hypothesis H_0. Wald improved this method by stating that in previous tests, when the value of λ_n was around C, the acceptance or rejection of the two hypotheses was judged too decisively, which appeared unreasonable. The improvement of Wald is to reject H_1 when λ_n is much smaller than C, accept H_1 when λ_n is much larger than C, and not give a conclusion when λ_n is very close to C. Instead, observe again and continue to detect λ_{n+1} until the likelihood ratio is much larger or smaller than C in the subsequent steps. In the sequential probability ratio test, the calculated likelihood ratio is compared with thresholds A and B to identify whether the spare parts are qualified. If the probabilities α and β of two types of errors are given in advance, then A and B can be calculated. The calculation formula for threshold sum is as follows:

$$A = \frac{1-\beta}{\alpha} \tag{14}$$

$$B = \frac{\beta}{1-\alpha} \tag{15}$$

The implementation steps for the verification are as follows: assuming x_1 is the value obtained from the first observation, calculate $\lambda_1(x)$, and if $\lambda_1(x) \le B$, stop the observation and accept the null hypothesis H_0; If $\lambda_1(x) \ge A$, stop observing and accept alternative hypothesis H_1; If $A < \lambda_1(x) < B$, extract the next group for testing, repeat the above steps until the requirements for stopping the testing are met, and finally give a judgment.

3. Conclusions

This paper aims to design a sampling inspection-based decision scheme to evaluate the proportion of defective parts in enterprise procurement as accurately as possible with the minimum number of inspection times. In the study, the authors used the binomial distribution to construct a sampling inspection model and analyzed the minimum sample size requirements at different confidence levels through the confidence interval. The results show that the minimum inspection sample size is 104 when the defective rate is 0.199 and the confidence level is 95%, and 335 when the defective rate is 0.063 and the confidence level is 90%. To further optimize the decision, the authors adopted the

Sequential Probability Ratio Test (SPRT), which allows for a significant reduction in experimental times while maintaining the same reliability.

In summary, this paper provides an efficient and low-cost sampling inspection scheme for enterprises to deal with uncertain defect rates, which can ensure the reliability of decision-making while minimizing the waste of inspection resources, providing a basis for cost control in production.

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