Optimal Decision-making for Production Processes Based on Goal Programming Algorithm

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Abstract. Quality inspection is the focus of global attention, related to the survival and development of enterprises. Focusing on the problem of product quality inspection, this paper proposes to design a reasonable sampling inspection scheme for enterprises under given conditions and formulate an optimal production inspection scheme in the industrial process of multi-process and multi-parts to maximize profits. By establishing Bayesian optimization model, quantifying decision variables and adopting goal planning algorithm, the optimal decision scheme under complex production environment is solved. However, in order to improve the applicability and accuracy of the model, the subsequent work should also include the dynamic assessment of the disassembled parts, and the dynamic adjustment of the market price and compensation. What's more, this model has a wide range of applicability and can help manufacturers in different fields to improve production efficiency and product quality while reducing production costs.

Keywords: Bayesian Optimization, Optimization Model, Product Quality Inspection.

1. Introduction

Quality inspection is a technical operation [1]. By inspecting and verifying products or services, it is determined whether they meet pre-set quality standards and specifications [2]. In today's society, especially in the context of global market competition, product quality issues have always been the focus of attention of consumers in various countries and are one of the key factors for the survival and development of enterprises. Whenever product quality is mentioned, especially in 3d scenes [3, 4] many people will immediately think of "Made in Germany [5]." From a large car to a small glass of beer, "German quality" is renowned globally. Fundamentally, it is inseparable from the strict control of commodity quality. Since China proposed "Made in China 2025 [6]" and Industry 4.0 [7] has entered a new era of Sino-German cooperation, China is committed to manufacturing transformation. Product quality is an important issue that needs to be focused on in the manufacturing process. Only by doing quality inspections can we promote the continuous upgrading of China's manufacturing industry to high-end, intelligent, and green levels, and steadily move towards becoming a manufacturing powerhouse.

In the process of product quality inspection, as long as one part is unqualified, the finished product is unqualified. Even if all parts are qualified, the finished product may not be qualified. For unqualified finished products, they can be scrapped or disassembled. To reduce costs, increase profits, and improve product quality, factories need to comprehensively consider inspection costs and customer satisfaction. Therefore, how to reasonably conduct sampling inspection processes is a major issue that enterprises need to consider. Thus, this paper presents the first research question (RQ):

RQ1: Under the conditions of given nominal values and reliabilities, how to design a sampling inspection plan with as few inspection times as possible for enterprises.

By quantifying the specific process of sampling inspection, this study obtains the optimal inspection plan, which enables enterprises to improve the accuracy of the defective rate obtained from sampling inspection while minimizing the number of samplings as much as possible. To achieve this, this paper simulates and quantifies the relevant values in quality inspection, conducts hypothesis testing according to known parameters, establishes a mathematical model and calculates, and finds the sampling plan with the minimum inspection cost.

With the complexity of production processes and the refinement of inspection processes, another key problem arises: In the production process with multiple processes and multiple parts, how to make decisions on product inspection in the process. Considering that different fixed costs (such as initial inspection costs) and variable costs (such as losses caused by defective products and the increased part assembly costs after disassembling finished products) will all have an impact on decision-making, multiple optimizations are needed. This is the RQ2 of this paper:

RQ2: How to formulate the optimal sampling inspection plan according to different product production and inspection processes to maximize profits.

According to different production processes, this paper quantifies the decisions at each stage into 0-1 variables, constructs a mathematical model of total profit and total cost, and takes maximizing total profit as the goal. Based on the goal programming algorithm [8], it simulates the entire process of production, sales, disassembly of unqualified products, and part reuse [9], and calculates the optimal decision-making plan [10].

By using mathematical formulas, algorithm models, and computational simulations, this paper aims to find the best solution for quality inspection in industrial production. The main contributions are: (1) Establish a Bayesian optimization model to provide a sampling inspection plan with as few samplings as possible. (2) According to the production inspection process, quantify the decisions at each stage into 0-1 variables, use the goal programming algorithm to construct a mathematical model of total profit and total cost to maximize profits. (3) Comprehensively consider the logical relationships between various processes, optimize the objective function, and then use the enumeration method to solve the specific optimal decision-making plan according to different constraint conditions.

2. Problem Formulation

Enterprises need to evaluate whether to accept this batch of parts provided by suppliers according to the sampling inspection method. When it is known that the defective rate of parts from suppliers will not exceed the nominal value, test protocols for rejecting this batch of parts when it is determined that the defective rate of parts exceeds 10% at a confidence level of 95% and for accepting this batch of parts when it is determined that the defective rate of parts does not exceed 10% at a confidence level of 90% are respectively given, and the number of tests is minimized while meeting the conditions.

Based on the defective rates of different parts, semi-finished products and finished products, as well as comprehensive data such as purchase unit price, inspection cost, market selling price, replacement loss, and disassembly cost, the author will construct a comprehensive decision-making model to comprehensively evaluate the entire production chain. This model will include several key stages, including whether to inspect parts; whether to inspect semi-finished products; whether to inspect finished products; and whether to disassemble unqualified finished products. In addition, for unqualified products, the company will unconditionally replace them. This paper will study two special cases (production environments consisting of 1 production process and 2 parts, and 2 production processes and 8 parts respectively), establish optimal decisions, and maximize total profit on the basis of meeting various indicators. Through this method, this research aims to provide specific process implementation plans to improve product quality and production profits.

3. Methodology

In this section, the process of building the models for Problems 1 and 2 will be elaborated in detail.

3.1. The establishment of Bayesian optimization model (RQ1)

In question 1, the company needs to decide whether to accept or reject the parts based on the supplier's claim that the defective rate will not exceed the nominal value $p_0 = 10\%$ and the sampling test results. Because the testing cost is borne by the enterprise itself, but too few testing times will lead to low reliability, so the enterprise needs to reasonably design sampling testing scheme to determine the number of sampling tests. If the product defect rate is p, when the total number of parts sampled for testing is i, since there are only two possibilities for a product: defective or non-defective, the number of defective products in sampling testing j follows the binomial distribution of parameter(i,p) , that is, $j \sim B(i,p)$. Considering that the quantity of spare parts produced by the supplier in the practical problem is sufficiently large, we can see from the *de Movire-Laplac*e central limit theorem: $\frac{j-i\cdot p}{\sqrt{i\cdot p\cdot(1-p)}} \xrightarrow{d} N(0,1)\ (i \to +\infty)$ and $\frac{j/i-p}{\sqrt{p\cdot(1-p)/i}} \xrightarrow{d} N(0,1)\ (i \to \infty)$. According to the hypothesis conditions, we can know from the knowledge of hypothesis testing that this is a one-sided hypothesis testing problem, we can assume the null hypothesis $H_0: p \le p_0$ and the alternative hypothesis $H_1: p > p_0$. In case (1), if it is determined with 95% confidence that the defective rate of parts exceeds the nominal value 10%, the lot of parts will be rejected. When H_0 is true, $\frac{\frac{j}{i}-p_0}{\sqrt{p_0\cdot(1-p_0)/i}}$follows the standard normal distribution $N(0,1)$, and the significance level $\alpha = 0.05$ is known by 95% confidence, and $\alpha = P\{reject\ H_0|H_0\ is\ true\} = P\left(\frac{\frac{j}{i}-p_0}{\sqrt{p_0\cdot\frac{1-p_0}{i}}} > C\right) = 1 = \phi(C)$ is obtained according to the definition of α. The critical value is obtained as $C = \phi^{-1}(1-\alpha)$, where ϕ^{-1} is the inverse function of the standard normal distribution, that is, when $\frac{\frac{j}{i}-p_0}{\sqrt{p_0\cdot\frac{1-p_0}{i}}} > C$, the batch of parts is rejected. Because of the cost of detection, when the actual $\frac{\frac{j}{i}-p_0}{\sqrt{p_0\cdot\frac{1-p_0}{i}}}$ does not fall into the rejection domain, this paper sets a higher penalty cost in order to judge whether the reliability requirements are met. Similarly, in case (2), a batch of parts is received if it is determined with 90% confidence that the defective rate of the parts does not exceed the nominal value. When H_0 is true, $\frac{\frac{j}{i}-p_0}{\sqrt{p_0\cdot\frac{1-p_0}{i}}}$ follows the standard normal distribution $N(0,1)$, and the significance level $\beta = 0.1$is known by 90% confidence, $\beta = P\{reject\ H_0|H_0\ is\ true\} = P\left(\frac{\frac{j}{i}-p_0}{\sqrt{p_0\cdot\frac{1-p_0}{i}}} > T\right) = 1 - \phi(T)$ according to the definition of β. The critical value is obtained as $T = \phi^{-1}(1-\beta)$, where ϕ^{-1} is the inverse function of the standard normal distribution, that is, when $\frac{\frac{j}{i}-p_0}{\sqrt{p_0\cdot\frac{1-p_0}{i}}} > T$, the batch of parts is rejected. That is, when $\frac{\frac{j}{i}-p_0}{\sqrt{p_0\cdot\frac{1-p_0}{i}}} \le T$, the batch of parts is received.

Due to the lack of actual defective rate, the condition can only obtain an inequality between the sample size and the actual defective rate, which cannot be described by an accurate mathematical formula. Bayesian optimization has no rigid requirements on the function, and the optimal decision of the function can be inferred only through a small number of sampling points. Therefore, this paper chooses to build a Bayesian optimization model.

The goal of the given problem is to find the minimum cost of detection under the reliability constraint, that is, the minimum number of detections. We established a Bayesian optimization model, tried to take different sample size i_0 to calculate the detection cost for many times, then used Gaussian process to fit the current detection results and predict the new i_0 , and used Gaussian process to simulate repeated iterations until the optimal sample size i satisfying the reliability constraints was found.

3.2. The establishment of Bayesian optimization model (RQ2)

3.2.1. Situation 1

Situation 1 is a production environment composed of 1 production process and 2 spare parts. The goal of this problem is to maximize total profit, which requires a balance between inspection costs and a potential increase in defective rates.

For the inspection decision, three 0-1 variables x, y and z are defined, which represent whether to test the parts and finished products respectively. Specifically, for each part and finished product, set the variable value to 0 (no detection) and 1 (detection). For the disassembly decision, it is regarded as a scale problem and simplified to define integer variable t, and $t/10$ is used to represent the disassembly proportion of unqualified products.

In order to consider the reuse of spare parts after disassembly of unqualified finished products, this paper needs to simulate the production process several times. Let the profit calculated in the k simulation be $g(k)$, and the objective function be the total profit $G = \sum_{i=1}^{k_m} g(i)$. Set the defective rate of parts 1, parts 2 and assembled finished products as r_1, r_2, r_3, the purchase unit price and inspection cost of parts 1 as $a_1\ and\ b_1$, the purchase unit price and inspection cost of parts 2 as $a_2\ and\ b_2$, the assembly cost, inspection cost and market selling price of finished products as $c_1, c_2\ and\ c_3$, and the replacement loss and dismantling cost of unqualified finished products as $d_1\ and\ d_2$, respectively. The number of parts is l.

The following mathematical models can be established:

The profit from the first simulation is:

$$g(1) = e_5 - e_1 - e_2 - e_3 - e_4 - e_6 - e_7, \tag{1}$$

Where, e_1 is the purchase cost of spare parts:

$$e_1 = l \cdot a_1 + l \cdot a_2, \tag{2}$$

e_2 Is the testing cost of spare parts?

$$e_2 = l \cdot b_1 \cdot x + l \cdot b_2 \cdot y, \tag{3}$$

e_3 Is the assembly cost of the finished product?

$$e_3 = (min\{l - l \cdot r_1 \cdot x, l - l \cdot r_2 \cdot y\}) \cdot c_1, \tag{4}$$

e_4 Is the testing cost of the finished product?

$$e_4 = (min\{l - l \cdot r_1 \cdot x, l - l \cdot r_2 \cdot y\}) \cdot c_2 \cdot z, \tag{5}$$

e_5 Is the selling price of the finished product?

$$e_5 = min\begin{Bmatrix} min\{l - l \cdot r_1 \cdot x, l - l \cdot r_2 \cdot y\} \cdot (1 - z), \\ min\{l - l \cdot r_1 \cdot x, l - l \cdot r_2 \cdot y\} \cdot (1 - r_3) \cdot z \end{Bmatrix} \cdot c_3, \tag{6}$$

e_6 Is the loss of replacement of unqualified finished products?

$$e_6 = \{l \cdot x \cdot y \cdot r_3 + l \cdot (1 - x \cdot y) \cdot [1 - (1 - r_1) \cdot (1 - r_2) \cdot (1 - r_3)]\} \cdot d_1, \tag{7}$$

And e_7 is the dismantling cost of unqualified finished products:

$$e_7 = \{l \cdot x \cdot y \cdot r_3 + l \cdot (1 - x \cdot y) \cdot [1 - (1 - r_1) \cdot (1 - r_2) \cdot (1 - r_3)]\} \cdot d_2 \cdot \frac{t}{10}. \tag{8}$$

In addition, $\frac{t}{10}$ is the proportion of nonconforming products returned by disassembled customers?

3.2.2. Situation 2

Situation 2 is the production environment consists of 2 production processes and 8 spare parts. This problem needs to be based on the assembly process of two processes and eight parts, according to the known parts, semi-finished products and finished products defective rate, inspection cost, assembly cost, disassembly cost, etc., to make the optimal decision scheme.

To facilitate data reference, the given conditions are abstracted. The defective rate of spare parts, purchase price and inspection cost are $P_i,\ A_i,\ B_i(i=1{,}2,\ldots{,}8)$, the defective rate of semi-finished products, assembly cost, inspection cost and disassembly cost are $R_i, C_i, D_i, E_i(i=1,2,3)$, and the defective rate of finished products, assembly cost, inspection cost, disassembly cost, market selling price and replacement loss are $P_f, F_1, F_2, F_3, F_4, F_5, F_6$. Let the total quantity of qualified finished products sold be $L(k)$. By analyzing the meaning of the question, for the same kind of parts, semi-finished products and finished products, the inspection and disassembly are all or none. Therefore, 16 logical variables $M_i(i=1,\ 2,\ldots,16)$ are set, the values of which are 0 or 1, where $M_1{\sim}M_8$ respectively indicates whether the parts 1~8 are tested, $M_9{\sim}M_{11}$ respectively indicates whether the semi-finished products 1~3 are tested, M_{12}indicates whether the finished products are tested, and M_{13} indicates whether the finished products are dismantled. $M_{14}{\sim}M_{16}$ Respectively indicates whether semi-finished products 1 to 3 are disassembled. Set $h_1(k)$ as part purchase cost, $h_2(k)$ as part inspection cost, $h_3(k)$ as semi-finished product assembly cost, $h_4(k)$ as semi-finished product inspection cost, $h_5(k)$ as semi-finished product disassembly cost, $h_6(k)$ as finished product assembly cost, $h_7(k)$ as finished product inspection cost, $h_8(k)$ as replacement cost of nonconforming product, and $h_9(k)$ as replacement cost of nonconforming product.

Based on the set conditions and assumptions, this paper lists the total cost calculation formula$H(k)=\sum_{i=1}^{9} h_i(k)$, where the quantity is:

$$L(k)=\begin{cases} N & when\ k=1 \\ N-l_2\cdot(k-1) & when\ k\geq 2 \end{cases}, \tag{9}$$

Parts purchase cost is:

$$h_1(k)=\sum_{i=1}^{8}\left(\frac{L(k)}{1-P_i\cdot(1-M_i)}-l_2(k-1)\right)\cdot A_i\,, \tag{10}$$

The cost of parts inspection is:

$$h_2(k)=\sum_{i=1}^{8}\frac{M_i\cdot L(k)\cdot B_i}{1-P_i}, \tag{11}$$

The assembly cost of the finished product is:

$$h_3(k)=3\cdot C\cdot L(k), \tag{12}$$

The cost of semi-finished product inspection is:

$$h_4(k)=L(k)\cdot D\cdot(M_9+M_{10}+M_{11}), \tag{13}$$

The pass rate of semi-finished product 1 is:

$$t_1=\prod_{i=1}^{3}\big(1-P_i\cdot(1-M_i)\big)\cdot(1-R)\,, \tag{14}$$

The pass rate of semi-finished product 2 is:

$$t_2=\prod_{i=4}^{6}\big(1-P_i\cdot(1-M_i)\big)\cdot(1-R)\,, \tag{15}$$

The pass rate of semi-finished product 3 is:

$$t_3 = \prod_{i=7}^{8} \big(1 - P_i \cdot (1 - M_i)\big) \cdot (1 - R)\,, \tag{16}$$

The quantity of unqualified and dismantled semi-finished products 1 is:

$$q_1(k) = L(k) \cdot M_9 \cdot t_1, \tag{17}$$

The number of unqualified and dismantled semi-finished products 2 is:

$$q_2(k) = L(k) \cdot M_{10} \cdot t_2, \tag{18}$$

The number of unqualified and dismantled semi-finished products 3 is:

$$q_3(k) = L(k) \cdot M_{11} \cdot t_3, \tag{19}$$

The dismantling fee of semi-finished products is:

$$h_5(k) = \big(q_1(k) + q_2(k) + q_3(k)\big) \cdot E, \tag{20}$$

The number of parts that can be used after dismantling the semi-finished product is:

$$l_1(k) = \frac{3 \cdot q_1(k) + 3 \cdot q_2(k) + 2 \cdot q_3(k)}{8}, \tag{21}$$

The qualified rate of finished products is:

$$t_4 = \prod_{i=1}^{8} \big(1 - P_i \cdot (1 - M_i)\big) \cdot (1 - R)^3 \cdot \Big(1 - P_f \cdot (1 - M_{12})\Big), \tag{22}$$

Finished product assembly cost:

$$h_6(k) = L(k) \cdot F_1, \tag{23}$$

Finished product inspection cost:

$$h_7(k) = M_{13} \cdot L(k) \cdot F_2, \tag{24}$$

The replacement cost of nonconforming products is:

$$h_8(k) = (1 - M_{12}) \cdot L(k) \cdot (1 - t_4) \cdot F_5, \tag{25}$$

The dismantling cost of the nonconforming product is:

$$h_9(k) = M_{13} \cdot L(k) \cdot (1 - t_4) \cdot F_3, \tag{26}$$

And the number of parts that can continue to be used after disassembly is:

$$l_2(k) = M_{13} \cdot L(k) \cdot (1 - t_4) + l_1(k). \tag{27}$$

4. Results

4.1. The result of RQ1

The parameter space is set, and a positive integer from 10 to 1000 iterations is taken. The Gaussian process optimization algorithm is used to find the parameter combination that minimizes the objective function by calling the objective function for a maximum of 50 times in the given parameter space.

According to FIG. 1 and FIG. 2, under two different conditions, the minimum sampling cost is obtained when the sample size is 162 and 104 respectively.

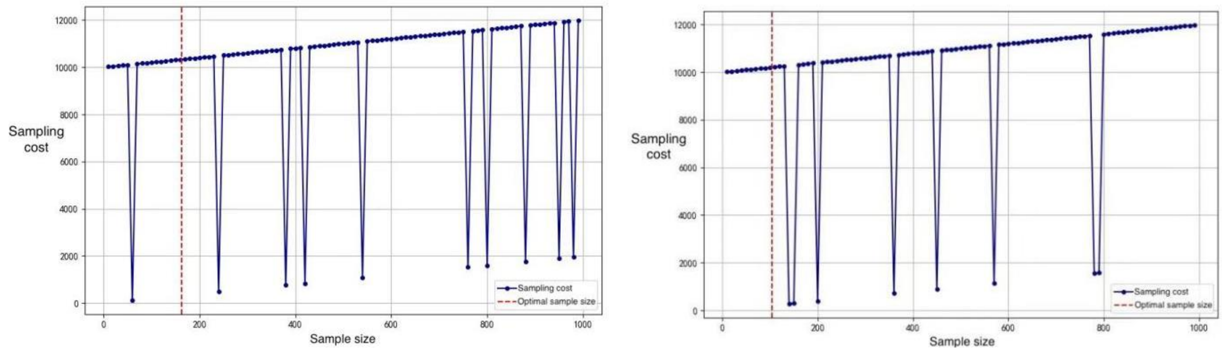

Figure 1. Relationship between sampling cost and sample size in situation 1 (Left) and 2 (Right).

4.2. The result of RQ2

For situation 1 (production environments consisting of 1 production process and 2 parts):

Let the initial quantity *l* of parts 1 and 2 both be 100. For the following six different cases in table 1, this paper finds the optimal decision.

Table 1. The cases encountered by enterprises in production.

Cases	Parts 1			Parts 2			Finished products				Defective finished products	
	Defective percentage	Purchase price	Inspection cost	Defective percentage	Purchase price	Inspection cost	Defective percentage	Assembly cost	Inspection cost	Market price	Replacement loss	dismantling cost
1	10%	4	2	10%	18	3	10%	6	3	56	6	5
2	20%	4	2	20%	18	3	20%	6	3	56	6	5
3	10%	4	2	10%	18	3	10%	6	3	56	30	5
4	20%	4	1	20%	18	1	20%	6	2	56	30	5
5	10%	4	8	20%	18	1	10%	6	2	56	10	5
6	5%	4	2	5%	18	3	5%	6	3	56	10	40

The following figure shows the specific decisions and maximum profit of the optimal decision in six cases:

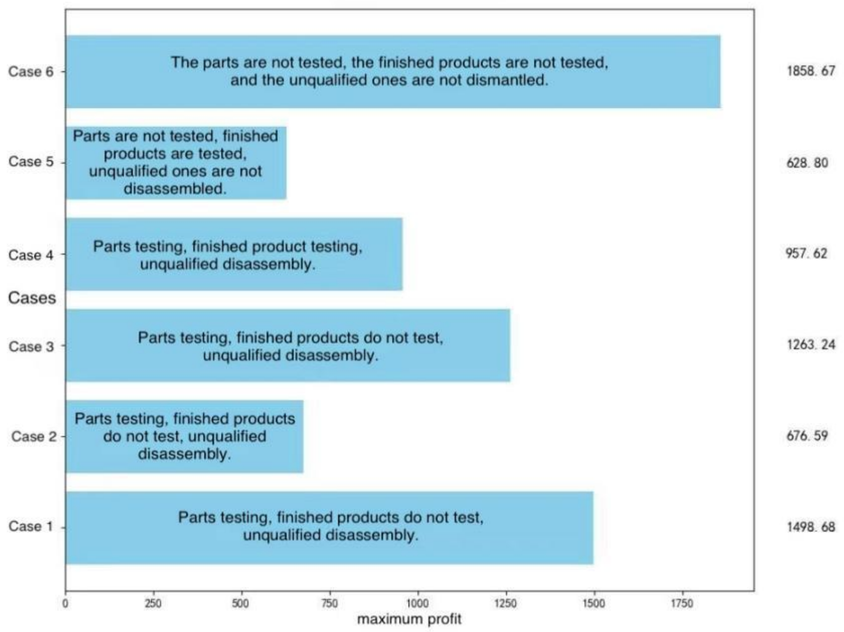

Figure 2. The maximum profit under the optimal decision in six cases.

For situation 2 (production environments consisting of 2 production processes and 8 parts):

To ensure that all customers who purchase products ultimately receive qualified products, we stipulate that N must be a fixed value, and now we specify $N = 1000$.

The following table (table 2) shows the values of cost, loss, selling price and defective rate in the assembly process.

Table 2. The cases encountered by enterprises in production.

Parts	Defective percentage	Purchase price	Inspection cost	semi-finished product	Defective percentage	Assembly costs	Inspection cost	Dismantling cost
1	10%	2	1	1	10%	8	4	6
2	10%	8	1	2	10%	8	4	6
3	10%	12	2	3	10%	8	4	6
4	10%	2	1					
5	10%	8	1	Finished product	10%	8	6	10
6	10%	12	2					
7	10%	8	1		Market price		Replacement loss	
8	10%	12	2		200		40	

According to the results of the code running, this paper can obtain that the total cost is minimized for all parts and finished products, no inspection for all semi-finished products, no disassembly for all defective semi-finished products and finished products. The minimum total cost is 98222.2 yuan.

5. Conclusions

To sum up, this paper solves the problem of designing a reasonable sampling inspection scheme for enterprises under given conditions and formulating an optimal production inspection scheme in the industrial process with multiple processes and multiple parts to maximize profits. By establishing a Bayesian optimization model, the most reasonable sampling times under different reliability conditions are obtained. Then, according to the production testing process, the decision variables of each stage are quantified, and the mathematical model of total profit is constructed by using the goal programming algorithm to optimize the objective function, and then the optimal decision scheme is solved by using the exhaustive method. According to different known conditions and production environment, the optimal decision scheme is different. The model proposed in this paper has wide applicability and can be applied to many manufacturing fields such as electronic products and machinery manufacturing. Enterprises can adjust the model parameters according to their own specific needs and conditions to improve production efficiency and product quality, while reducing costs. To ensure the reliability and effectiveness of decision-making, future work should also include dynamic assessment of defective parts after dismantling, as well as real-time adjustment of prices and losses. In addition, the introduction of robustness analysis can also enhance the stability and optimization effect of the model in the face of market fluctuations and uncertainties.

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