# Production Process Optimization and Defective Rate Control Based on Adaptive Sequential Detection and Monte Carlo Tree Search

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**Abstract.** With the increasing globalization and complexity of supply chains, modern manufacturing faces challenges in ensuring product quality while minimizing inspection costs. Companies must balance the need for reliable components from diverse suppliers with the risks of defective parts that could disrupt production. Traditional sampling methods often lack flexibility, leading to inefficiencies in defect detection. To address these challenges, this study integrates adaptive sequential detection with Monte Carlo Tree Search (MCTS) to optimize production processes. The adaptive detection approach adjusts sample sizes dynamically, ensuring defective parts are rejected with 95% confidence or accepted with 90% confidence, maintaining precision while controlling costs. When the defective rate nears 0.11, the method requires up to 155 samples for accurate decisions. MCTS further enhances decision-making by simulating multiple production strategies and identifying the optimal path, balancing inspection costs with quality control. Among the six tested scenarios, the best-case strategy achieved an expected return of 47.2 units, highlighting the model's effectiveness. This framework provides a practical solution for manufacturers to improve decision-making under uncertain conditions and can be extended to multi-stage production lines, offering valuable insights for addressing supply chain disruptions and complex operational challenges.

**Keywords:** Adaptive Sequential Detection, Monte Carlo Tree Search, Inspection Costs, Production Quality Control, Supply Chain Management.

## 1. Introduction

In the modern manufacturing industry, the production process of enterprises is increasingly complex, especially in the supply chain management and production quality control is facing many challenges. With globalization and the continuous extension of supply chains, companies not only have to deal with diverse suppliers but also need to ensure that the quality of spare parts provided by each supplier is stable and reliable. However, the quality problems of spare parts, especially the high defective rate, may lead to the stagnation of the entire production line, and even have a serious impact on the quality of the final product. Therefore, how to effectively identify and deal with defective parts without increasing the cost of inspection has become a key issue in the quality management of enterprises.

Although the traditional sampling detection method can identify a certain proportion of defective products, the traditional sampling method is often designed according to the preset fixed sample size [1]. This unified sampling strategy lacks adaptability to specific situations, which may lead to the sample size is not optimal in practical application, that is, there may be excess or insufficient sample size [2]. This study used normal approximation to estimate the distribution of defective rates, adopted an adaptive sequential detection method to set sampling targets and initial parameters, conducted several experiments, and finally obtained the required number of detections under different actual defective rates. The method of dynamic programming combined with Monte Carlo tree search is used to explore all possible paths, find the optimal path combination of the whole production process, and finally get the best decision for each stage.

## 2. Design and Analysis of Adaptive Sequential Sampling Inspection Scheme

### 2.1. The solution of adaptive sequential sampling detection scheme

Data in this study are derived from www.mcm.edu.cn. The scenario of this study is that the supplier claims that the defective rate of a batch of parts will not exceed the nominal value (10%). Enterprises need to provide testing schemes for two situations: one is to refuse to receive the batch of parts if the defective rate exceeds the nominal value when the reliability is 95%; The second case is to receive the batch of parts if the defective rate does not exceed the nominal value under the condition of 90% reliability [3]. This study first established a binomial distribution model combined with normal approximation. Considering that there are likely to be some close situations in real life, for example, the calculated result is very close to 95%, between 94% and 96%, in this case, the adaptive sequential detection scheme chooses to increase the number of samples to re-sample detection [4, 5]. This enables companies to accurately make different decisions about the two confidence levels in dynamic changes [6].

### 2.2. Solution idea of adaptive sequential sampling detection scheme

Step1 Determine the sampling target and initial parameters:

The goal of sampling testing is to know whether the defective rate of this batch of parts purchased from the supplier is greater than 10%, which requires a scheme to be designed and the minimum number of sampling times required to judge. The initial value is set, and the defective rate of the sample is 10%, that is $P_0 = 0.10$, the initial defective rate is $n_0$, and the initial number of samples $n_0 = 30$, which is generally set to 30 to 50. Here, we set the initial sample, and determine the standard normal critical value corresponding to 95% confidence degree $Z_{0.025} = 1.96$, and the standard normal critical value corresponding to 90% confidence degree$Z_{0.05} = 1.645$,

Step 2 Conduct initial sampling.

First of all, calculate the defective rate with the initial sample $n_0 = 30$, assuming that 4 defective products are detected in the initial sample, then the defective rate is estimated as:

$$\hat{p} = \frac{4}{30} = 0.1333 \tag{1}$$

After obtaining the defective rate, the confidence interval of the defective rate is calculated. Here, the approximate formula of normal distribution is used to calculate the confidence interval:

$$\hat{p} \pm Z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \tag{2}$$

Where n is the sample size and P is the estimated defective rate. Taking 95% confidence as an example, the confidence interval can be calculated as (0.0129, 0.2537).

Step 3 Decide to reject or accept according to the confidence interval solved in the previous step.

The lower limit of the 95% confidence interval calculated in the previous step is 0.0129, which is less than 10%, so the rejection judgment cannot be made, and it is necessary to continue sampling. Similarly, if the upper limit of the 90% confidence interval in this example does not exceed 10%, and therefore the acceptance judgment cannot be made for the time being, it is also necessary to continue sampling. After the decision to continue sampling is made, the sample number should be updated. In this model, we adopt the method of gradually increasing the sample number to update the sample number and repeat steps 2 and 3.

Step4 Stop sampling when the results obtained in Step 3 do not need to continue updating the sampling and make the final decision based on the sample size.

### 2.3. Model solving and analysis of adaptive sequential sampling detection scheme

Using Python to implement the above adaptive sequence model, the results obtained with 90% and 95% confidence respectively are shown in Figure 1 and Figure 2.

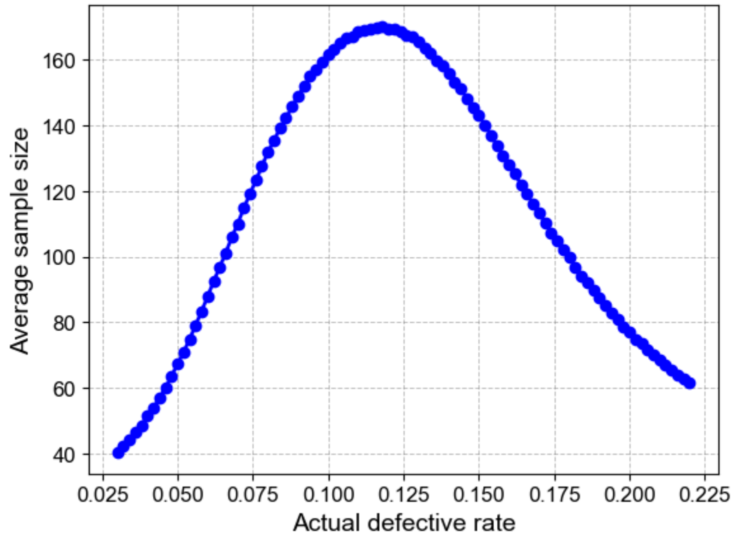


**Figure 1.** Average sample size with 95% confidence and analysis

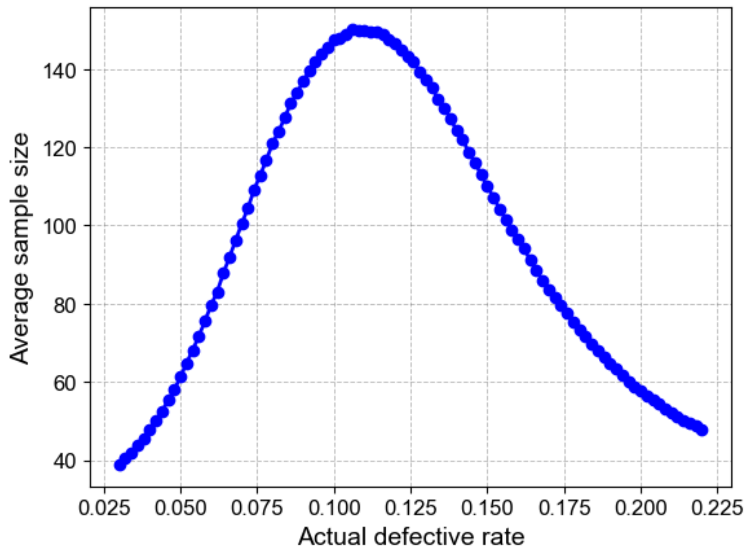


**Figure 2.** Average sample size with 90% confidence and analysis

It can be seen from Figure 1 and Figure 2 that the trend of the images in both cases is similar to that of the quadratic function, and 100,000 tests have been conducted in both cases, and the results are accurate and reliable. As can be seen from the two charts, the average number of samples required is the highest when the actual defective rate is about 0.11. This is because when the actual defective rate is close to the nominal defective rate, the system is difficult to quickly decide whether to accept or reject, at this time, it is necessary to rely on more samples to further narrow the gap to determine whether to accept or reject.

When the reliability is 95%, the results obtained are shown in Figure 1. We can roughly see that when the defective product rate provided by the supplier ranges from 0.03 to 0.22, the average minimum sampling quantity is about 38, the defective product rate is about 0.028, the average maximum sampling quantity is about 155, and the actual defective product rate is 0.11. Enterprises can decide the number of sampling times according to the figure provided in Figure 2. For example, when the defective rate is about 0.050, the average sampling number can be designed to be 60.

Meanwhile, when the defective rate provided by the supplier is about 0.070 to 0.16, the average sampling number is about 100 pieces, which is a large number, and enterprises should consider this situation when considering the testing cost.

When the reliability is 90%, the results can be roughly obtained as shown in Figure 2. When the defective product rate provided by the supplier ranges from 0.03 to 0.22, the average minimum sampling quantity is about 40, the defective product rate is about 0.028, the average maximum sampling quantity is about 170, and the actual defective product rate is 0.11. The range of the average sample size in this case is larger than that in the first case, the trend is roughly the same, and the proportion of the average sample size greater than 100 is larger than in the first case. Enterprises can decide the number of sampling times according to the figure provided in Figure 3. For example, when the defective product rate is about 0.2, the average sampling number can be designed to be 80. Meanwhile, when the defective product rate provided by the supplier is about 0.065 to 0.176, the average sampling number is about 100 pieces, which may have a greater impact on the testing cost. Companies should consider such situations when making decisions.

## 3. Optimal Decision Design with Dynamic Programming and Monte Carlo Tree Search

### 3.1. Thought analysis

The research will be divided into two parts, one is node, the other is global, to establish a model that can first solve the local optimal solution in each node, then calculate the global benefits, and choose the best production plan. Dynamic programming can calculate the optimal solution of each local decision point to ensure that the best choice can be made in each step of detection, assembly, disassembly, and other operations. After nodes are found, the Monte Carlo model is used to simulate the global path, and the Monte Carlo tree search is used to explore various possible strategy combinations and find the optimal solution for the overall path. The ability to find the optimal production decision [7].

### 3.2. Model building

Step 1 represents the state of each decision point.

The process contains four behaviors: detect, do not detect, disassemble, and discard, in which detect and do not detect are opposed, and disassemble and discard are opposed, so model 01 can be used to represent the state of each decision point:

$x_1$: Whether to detect parts 1 (0 indicates no detection, 1 indicates detection).

$x_2$: Whether to detect parts 2 (0 indicates no detection, 1 indicates detection).

y: Whether to test the finished product (0 means no test, 1 means test).

z: Whether to disassemble unqualified products (0 means no disassembly, 1 means disassembly).

Step 2 Apply the dynamic programming recursion formula

The idea of the dynamic programming recursion formula is to go through all the stages of the decision and calculate the benefits of all the decisions. The following is the dynamic programming recurrence relationship:

Parts inspection benefits: Seek a balance between inspection costs and defective rates

$$V(x_1, x_2) = max( _{Inspection\ Cost} \times p_1, _{Purchase\ Cost} \times (1 - p_1)) \tag{3}$$

$P_1$ Refers to the defective rate of parts 1, the calculation of parts 2 in the same way, that $P_2$ refers to the defective rate of parts 2, parts detection income in line with the binomial distribution, that is, either the detection by receiving, or the detection of unqualified rejection, the formula is to use the defective rate to calculate the detection cost and purchase cost expectations, and then take the maximum.

Product inspection revenue: Find a balance between the market price of the finished product and the rate of defective products.

$$V(y) = max\left( Market\ Selling\ Price \times \left(1 - p_{product}\right), Inspection\ Cost \right) \quad (4)$$

The disassembly income also satisfies the binomial distribution. This formula uses the defective rate of the finished product to calculate the disassembly cost and the expected disassembly cost respectively, and takes the maximum of them.

Step 3 Use dynamic programming recursion.

Using dynamic programming recursion, the sequence is from back to front, dividing the process into many stages, knowing that each stage is not affected by the previous process state, and the behavior of the current stage is only related to that stage. The local optimal solution may not constitute the global optimal solution, but the global optimal solution must contain some local optimal solutions. The order from back to front not only ensures that all nodes need to be calculated only once but also avoids redundant calculation, and the problem can be simplified step by step according to this orde[8]. The decision of each stage node will depend on the calculated optimal solution so that the global optimal solution is easier to find.

Therefore, the recursive process starts from the customer return stage to calculate the optimal decision and then returns to the finished product inspection stage to calculate whether to conduct the finished product inspection according to the cost and income of the inspection. Finally, it goes back to the parts inspection stage to determine whether to conduct the parts inspection according to the defective rate and inspection cost.

Step 4 combines dynamic programming with the Monte Carlo tree search.

Monte Carlo tree search (MCTS) is very advantageous in simulating many possible paths, especially when there is uncertainty in the problem, through a large number of simulations, the path with the highest expected return can be found. At each decision point, MCTS will conduct several simulations, update the expected return after each simulation, and finally choose the path with the highest return [9]. The result will be backpropagated to the previous node, ensuring that the global benefit is maximized [10]. In this case, the Montecarlo tree search can simulate the finished product inspection path, the parts inspection path, the disassembly or the discard path [11].

The path of each simulation can be represented as:

$$p = \{\text{Inspect Part 1, Inspect Part 2, Inspect Product, Disassemble Nonconforming Product}\} \quad (5)$$

It can simulate multiple decision combinations of entering the market or dealing with unqualified finished products to find the optimal path, and the total return of the path can be expressed as:

$$R(P) = \left(1 - p_{product}\right) \cdot Selling\ Price - \Sigma\ Cost\ Items \quad (6)$$

Dynamic programming can calculate the optimal solution of each independent decision point to ensure that the optimal choice is made in each step of operations. Monte Carlo tree search can simulate different global paths, explore a variety of possible strategy combinations, and find the global optimal solution. The combination of the two can not only ensure the correct processing at each stage but also find the path with the highest expected value in the global scope [12].

### 3.3. Result analysis

python is used to realize the model of dynamic programming combined with Monte Carlo tree search, and the following results are obtained according to the above solution process. We can get six combinations corresponding to the six situations encountered in the production of enterprises, and the list of program operation results is shown as follows (table 1):

**Table 1.** The decision of six situations encountered in the production of enterprises

| number | Whether to test parts 1 | Whether to test parts 2 | Whether to test parts 3 | Whether to test parts 4 | expectation |
|---|---|---|---|---|---|
| 0 | NO | NO | YES | YES | 36.4 |
| 1 | NO | YES | YES | YES | 30.8 |
| 2 | NO | NO | YES | YES | 36.4 |
| 3 | NO | YES | YES | YES | 31.8 |
| 4 | NO | YES | YES | YES | 37.4 |
| 5 | NO | NO | NO | NO | 47.2 |

The results show that parts 1 are not tested in all cases, finished products in cases 1-5 are tested, and cases 6 are not tested. The unqualified finished products in cases 1-5 are disassembled, and cases 6 are not disassembled, and then cases 1 and 3 are tested for parts 2. The income expectation of both cases is 36.4. The expected return is 30.8. The expected return is 31.8 and 37.4 respectively when parts 2 of cases 4 and 5 are tested. For case 6, there is no need to test and the expected return is 47.2.

## 4. Conclusion

This study proposed a novel approach to optimize the production process and control the defective rate by combining adaptive sequential detection and Monte Carlo Tree Search (MCTS). The adaptive sequential detection method dynamically adjusts sampling sizes based on real-time conditions, ensuring precise defective rate estimates with minimized inspection costs. This approach helps enterprises make informed decisions by balancing inspection reliability and cost-effectiveness.

Additionally, integrating MCTS enables us to identify optimal production paths by simulating various scenarios and strategies, ensuring that the global benefits are maximized across different stages of the manufacturing process. Through experimental validation, the proposed framework has demonstrated its feasibility and practical value. This study provides a research framework and methodology that can be extended to the fields of quality control and management, offering a robust solution for enterprises facing complex challenges in modern supply chains.

Future work could explore applying this model to more complex production environments, including multi-stage assembly lines and scenarios with varying supply chain disruptions, enhancing its adaptability and scalability in practical applications.

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