Research on sampling inspection and process optimization model in process production

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Abstract. In this paper, the decision problem in the production process of the company is studied, and the defective rate is evaluated by using the hypothesis testing model and Monte Carlo method. Assuming that the real defective rate and nominal value error are 0.05, the minimum sampling times are 50, 66, and 75 when the total capacity is 100, 200, and 300 with 95% confidence. The minimum number of samplings in the infinite population case is 98. The multi-stage decision-making model based on dynamic optimization is constructed, the value function and dynamic recursion are added to Markov decision, the production decision of each stage is analyzed and solved, the objective function of maximum profit is obtained, the corresponding production decision is determined, and the dynamic optimization is evaluated and analyzed by sensitivity analysis. Taking parts 1 as an example, comparing situation 2 and situation 4, it is found that the testing cost of parts and finished products has changed its maximum profit and corresponding decision, which is mainly reflected in the decision of whether to inspect parts and finished products. It is concluded that the detection cost has a great influence on the optimal decision.

Keywords: Product Testing, Product Decision, Hypothesis Testing, Dynamic Optimization, Multistage Decision.

1. Introduction

Lean manufacturing is a manufacturing method designed to minimize the waste of resources, with the goal of improving production efficiency, reducing costs, shortening lead times and ensuring product quality [1]. In today's fierce market competition, lean production has become an important tool for many enterprises to improve their competitiveness. However, implementing lean production is not an overnight task. Many companies encounter various challenges in practice. In addition, the data dilemmas, business challenges, and application issues that enterprises face in the decision-making process are also important areas of research [2]. There are multiple financial information systems in the enterprise, data sources are scattered, and the lack of unified and standardized data standards makes it difficult to communicate data. The lack of comprehensive financial indicators and data support for management decision-making, the lack of reductive drill-down analysis path and automated financial early warning mechanism makes it difficult to resolve risks in time. Without terminal applications such as large-screen and mobile terminals, a single PC report cannot meet the data viewing and analysis requirements of various financial users and in various scenarios [3].

This paper mainly studies the decision-making dilemmas faced by enterprises in the process of product manufacturing. This paper discusses how to construct the most reasonable decision-making model when two key components need to be purchased and used, so as to reduce the adverse impact of defects on enterprises while ensuring production quality, and assist in fine production cost management. Using optimized hypothesis testing model and dynamic optimization to help enterprises solve decision dilemma. Compared with the traditional hypothesis testing method, this method classifies and discusses different scenarios of population capacity, and builds different distribution models for small, finite and infinite populations respectively. It determines the minimum sample size needed to satisfy the confidence interval in both finite and infinite samples, ensuring that the sample is representative and enhancing the overall confidence of the product. In contrast to traditional static optimization models, dynamic optimization allows for real-time feedback and adjustments, such as process monitoring, to ensure that the production process remains at its best. This meets the actual needs of enterprises to adjust process parameters regularly in actual production scenarios.

2. Study on the defective rate of the company's products

2.1. Model Construction

In designing the sampling inspection scheme, this paper employs different statistical models and analytical methods to determine the minimum sample size based on varying total part population N. For small populations (N<30), a binomial distribution is used to analyze and determine rejection standards based on defect rate and the minimum sample size; for finite populations (30≤N<∞), a normal approximation is considered; for infinite populations (N=∞), a normal distribution is also used, with one-sided hypothesis testing and confidence interval methods to calculate the minimum sample size [4]. This scheme ensures that the required minimum sample size can be effectively determined under different confidence levels to optimize cost-efficiency. The nominal value set in the problem refers to the ideal, theoretical defect rate specified for parts in the production process, and this paper uses it as an evaluation standard for related solutions and optimizations [5].

2.2. Model Analysis

When the total part population N is less than 30, this paper uses a binomial distribution to calculate and determine the minimum sample size. The objective is to determine whether the defect rate p exceeds the nominal value $p_0 = 0.10$ (i.e., 10%) and to make an acceptance or rejection decision based on the sampling inspection data.

Using the cumulative distribution function (CDF) of the binomial distribution, this study calculates the probability of detecting a number of defects exceeding a certain threshold when the defect rate is 10%. The following inequality is used to determine the minimum sample size n:

$$P(X \geq k|\ n, p) = 1 - P(X < k|\ n, p) \leq 0.05 \tag{1}$$

Where $P(X < k|\ n, p)$ represents the probability that the number of defects is less than k under the conditions of sample size n and defect rate p.

Different values of n can be tested; by calculating the cumulative probability, the minimum sample size that meets the criteria can be identified. Due to the complexity of the calculations, this study conducts only qualitative analyses.

When the number of trials n is large and the probability of a single trial p is not very small, the normal distribution can approximate the binomial distribution. A rough rule for approximating the normal distribution is:

$$n \cdot p \cdot (1-p) \geq 9 \tag{2}$$

From the binomial distribution, the expected value is $\mu = np$ and the variance is $\sigma^2 = np(1-p)$. This study adopts the following correction method for approximation:

$$P(x_1 \leq X \leq x_2) = \underbrace{\sum_{x_1=k}^{x_2} \binom{n}{k} \cdot p^k \cdot q^{n-k}}_{EF} \approx \underbrace{\Phi\left(\frac{x_2 + 0.5 - \mu}{\sigma}\right) - \Phi\left(\frac{x_1 - 0.5 - \mu}{\sigma}\right)}_{ZF} \tag{3}$$

In this formula, $q = 1 - p$, EF refers to the binomial distribution, and ZFrefers to the normal distribution. This formula reflects the transition from the binomial distribution to the normal distribution. The upper (lower) critical values are adjusted by a correction factor of 0.5 to achieve more precise approximations when σ is large; this correction can be ignored when σ is tiny.

When the overall capacity is large (N > 30), this study considers using the normal distribution to approximate the binomial distribution. According to the central limit theorem, the binomial distribution $B(n, p)$ can be approximated by the normal distribution.

2.3. Infinite Population Model Solution

At a 90% confidence level, in scenarios where the acceptance of defective spare parts does not exceed the nominal value, companies wish to ensure that when the defect rate does not exceed 10%, they can accept that batch of spare parts.

The null hypothesis H_0: The defect rate $p \le 0.10$ (the defect rate of the spare parts does not exceed the nominal value of 10%) [6].

To perform hypothesis testing, this study sets the standard normal distribution test statistic Z. The calculation formula is:

$$Z = \frac{\overline{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \tag{4}$$

Where $\overline{p} = \frac{X}{n}$ is the true defect rate in the sample, $p_0 = 0.10$ is the nominal defect rate, n is the sample size, and $d = \overline{p} - p_0$ is the margin of error.

Using formula (7), we can derive:

$$n = \frac{Z_\alpha^2 \cdot p \cdot (1-p)}{d^2} \tag{5}$$

This study selects a significance level α, for example, 0.05 (the probability of incorrectly rejecting the null hypothesis is 5%). Based on the significance level α and the distribution of the test statistic Z, the paper determine the critical value Z_α or calculate the p-value. Under 90% confidence, the condition for accepting the spare parts is: $Z < Z_{0.90}$, where $Z_{0.90} = 1.28$ is the critical value.

Under 90% confidence, $Z = 1.28$, the equation yields:

$$n = \frac{(1.28)^2 \cdot 0.10 \cdot (1-0.10)}{0.05^2} = 60 \tag{6}$$

Thus, under 90% confidence, at least about 60 samples need to be drawn.

2.4. Finite Population Model Solution

In contrast to infinite populations, the total sample size for a finite population is limited. Based on the test statistic formula for infinite populations, this study proposes the following modifications for hypothesis testing calculations. The finite population correction formula is:

$$n' = \frac{n \cdot N}{n + N - 1} \tag{7}$$

N is the size of the population. If the population N is large, the finite population correction has a negligible effect and can be ignored.

Consequently, the sample size formula becomes:

$$n' = \frac{Z_\alpha^2 \cdot p \cdot (1-p)}{d^2} \tag{8}$$

This study presents the minimum testing sample sizes for different population capacities N of 100, 200, and 300 under 95% and 90% confidence levels, as shown in Table 1.

Table 1. Minimum test sampling times under different population capacities

N	With 95% confidence	With 90% confidence
100	50	38
200	66	46
300	75	50

For N=100N = 100N=100, the results indicate that the minimum testing sample size is 50 under 95% confidence and 38 under 90% confidence. The minimum testing sample sizes under 95% confidence are consistently greater than those under 90% confidence.

3. Dynamic optimization

3.1. Model Establishment

The experimental data are taken from one company, as shown in Table 2.

Table 2. Experimental data from one company

	Spare parts 1			Spare parts 2			Finished product				Defective product	
condition	defective	purchase	inspection	defective	purchase	inspection	defective	assembly	inspection	market	replacement	dismantling
1	10%	4	2	10%	18	3	10%	6	3	56	6	5
2	20%	4	2	20%	18	3	20%	6	3	56	6	5
3	10%	4	2	10%	18	3	10%	6	3	56	30	5
4	20%	4	1	20%	18	1	20%	6	2	56	30	5
5	10%	4	8	20%	18	1	10%	6	2	56	10	5
6	5%	4	2	5%	18	3	5%	6	3	56	10	40

In decision-making regarding process flows, whether to select each node depends on comparing the financial losses incurred by the process. According to Table 2, this includes whether to inspect spare parts, whether to inspect finished products, whether to disassemble defective items, whether to conduct exchanges, and the costs associated with purchasing spare parts and assembling them, all of which involve financial losses [7]. After removing defective items, the profits from selling finished products can be considered, allowing the establishment of a profit benefit model for finished products.

The total purchasing cost for parts 1 and 2 is:

$$L_{\text{Parts purchase cost}} = n_{\text{Spare parts 1}} \times L_{\text{Spare parts 1}} + n_{\text{Spare parts 2}} \times L_{\text{Spare parts 2}} \tag{9}$$

Where $n_{\text{Spare parts 1}}$ and $n_{\text{Spare parts 2}}$ are the quantities of spare parts 1 and 2 purchased, and $L_{\text{Spare parts 1}}$ and $L_{\text{Spare parts 2}}$ are their respective purchasing costs.

The total financial loss is:

$$L = L_{\text{Parts Inspection}} + L_{\text{Finished product testing}} + L_{\text{Disassembly}} + L_{\text{Swap}} + L_{\text{Assembly}} + L_{\text{Parts purchase cost}} \tag{10}$$

In this formula, $L_{\text{Parts Inspection}}$ is the financial loss from inspecting spare parts, $L_{\text{Finished product testing}}$ is the financial loss from inspecting finished products, $L_{\text{Disassembly}}$ is the financial loss from disassembling defective finished products, L_{Swap} is the unconditional exchange cost when defective items are returned by customers, and L_{Assembly} is the financial loss for assembling each part into a finished product.

The financial revenue from sales of qualified products is:

$$V_{\text{Sale}} = n_{\text{Qualified finished products}} \times m_{\text{Market price}} \tag{11}$$

The total profit benefit is:

$$V = V_{\text{Sale}} - L \tag{12}$$

Next, this study compares the financial losses incurred by two types of decisions made at each stage, with the comparison of financial losses limited to that specific phase.

The formula for inspection costs is given by:

$$L_{\text{Parts Inspection}} = n \times L_d \tag{13}$$

In this formula, n represents the total number of inspected spare parts, and L_d is the cost of inspecting each spare part, with the varying costs for different situations obtainable from a reference table.

Let the defect rate be $p_{\text{Defective}}$,while the defect rate after assembly is denoted as $p_{\text{Defective products after assembly}}$. The potential financial loss associated with the defective items involved in assembly is:

$$F_{\text{Loss 1}} = n \times p_{\text{Defective}} \times L_{\text{Assembly}} + n \times p_{\text{Defective}} \times p_{\text{Defective products after assembly}} \times L_{\text{Swap}} \tag{14}$$

When comparing financial losses, if $L_{\text{Parts Inspection}} < F_{\text{Loss 1}}$, the decision will be to inspect the spare parts.

If the decision is made to inspect the finished products, defective finished products can be identified, which may consist of a combination of qualified and unqualified parts. If finished product quality is not inspected, defective products may enter the market, leading to customer exchange and return losses.

The inspection cost for finished products is given by:

$$L_{\text{Finished product testing}} = n \times L_f \tag{15}$$

In this formula, L_f is the cost of inspecting each finished product. Let the defect rate in the finished products be $p_{\text{Finished product defect rate}}$, then the potential exchange loss is:

$$F_{\text{Loss 2}} = n \times p_{\text{Finished product inspection rate}} \times L_{\text{Swap}} \tag{16}$$

Thus, if $L_{\text{Finished product testing}} < F_{\text{Loss 2}}$, the decision will be to conduct finished product inspections.

For the defective products identified during inspection, a decision can be made on whether to disassemble them for investigation. If disassembly is performed, the parts can re-enter the production process. If there are qualified spare parts, they can be reused to generate qualified finished products, thus yielding revenue. However, disassembly incurs a cost, making it necessary to calculate whether the disassembly action leads to a financial loss or gain.

The disassembly cost for defective finished products is:

$$L_{\text{Disassembly}} = n_{\text{Failure}} \times L_s \tag{17}$$

In this formula, n_{Failure} is the number of defective finished products, and L_s is the disassembly cost for each finished product. If the disassembled qualified spare parts can generate revenue, let $p_{\text{Qualified parts}}$ represent the probability of obtaining qualified parts post-disassembly, and $V_{\text{Parts recycling}}$ be the revenue from each recoverable qualified spare part. The financial gain from disassembly can then be expressed as:

$$F_{\text{Dismantling benefits}} = n_{\text{Failure}} \times p_{\text{Qualified parts}} \times (V_{\text{Parts recycling}} - L_{\text{Assembly}}) \tag{18}$$

Therefore, when $L_{\text{Disassembly}} < F_{\text{Dismantling benefits}}$, disassembly is advantageous, and the decision will be to disassemble.

When defective finished products enter the market, companies will automatically provide exchange services. For returned finished products, a company can choose to classify them as defective and scrap them or disassemble the returned products. If no disassembly is performed, there are financial losses associated with exchanges, including logistics costs and reputational damage. If disassembly is chosen, the costs of return must be accounted for, alongside the disassembly costs, but the potential value from the qualified spare parts obtained through disassembly also contributes to revenue [8].

If the decision is to scrap, let $L_{\text{Logistics}}$ represent the logistics cost during the return process, then the financial loss is:

$$L_{\text{Return}} = L_{\text{Logistics}} \tag{19}$$

If the returned products still have value post-disassembly and disassembly is chosen, let $V_{\text{The recycling value of spare parts after disassembly}}$ denote the value of qualified spare parts that can be recovered from the disassembly of defective finished products, then the financial loss is:

$$L_{\text{Return for disassembly}} = L_{\text{Logistics}} + (L_{\text{Disassembly}} - V_{\text{The recycling value of spare parts after disassembly}}) \tag{20}$$

Given different production scenarios and phases, since each step of the process occurs sequentially, the total financial losses from all the aforementioned steps are summed up, leading to the total financial loss post-process decision:

$$L_{\text{Total capital loss}} = \sum_{i=1}^{n} (L_{\text{Parts Inspection}}(i) + L_{\text{Finished product testing}}(i) + L_{\text{Disassembly}}(i) + L_{\text{Return}}(i) + L_{\text{Assembly}}(i)) \tag{21}$$

In quality inspection decision-making, this study calculates relevant probabilities for each situation in a phased manner to optimize decisions, ultimately refining the production process structure [9]. The comprehensive decision model aims to maximize finished product profits while ensuring basic procurement costs and minimizing decision costs, thus maximizing the target total quantity V.

The objective function for the comprehensive decision model is:

$$V = V_{\text{Sale}} - L_{\text{Total capital loss}} - L_{\text{Parts purchase cost}} \tag{22}$$

3.2. Model Solution

The quality inspection process for production is divided into multiple stages, with multiple feasible solutions, each corresponding to a specific decision loss value. This study aims to find the optimal solution, maximizing finished product profit benefits across six different scenarios, thereby making optimal decisions [10]. Thus, a dynamic optimization method is applied to optimize the decision path for the finished product profit benefit model.

First, for a simple divide-and-conquer problem, the recursive equation for the Fibonacci sequence is:

$$F(n)=\begin{cases}1 & (n=0,1)\\ F(n-1)+F(n-2) & (n>1, n\in \mathrm{N}^{*})\end{cases} \tag{23}$$

Based on the value function defined in the Markov decision process, this study derives the recursive equation for dynamic optimization as:

$$V(s)=\max_{\beta}[V(s,\beta)+\sum_{s'}P(s'|s,\beta)V(s')] \tag{24}$$

In this formula, $V(s)$ is the loss function of state s, representing the maximum expected profit starting from state s; $V(s')$ is the profit function of state s'; $V(s,\beta)$ is the immediate profit from taking action β in state s; and $P(s'|s,\beta)$ is the state transition probability of moving from state s to state s' after taking action β. The final formula recursively sums across all layers to yield the decision with the maximum profit benefit.

The basis for the optimal scheme is to maximize the profit value of finished products, with the results of the finished product profit benefit model illustrated in Figure 1.

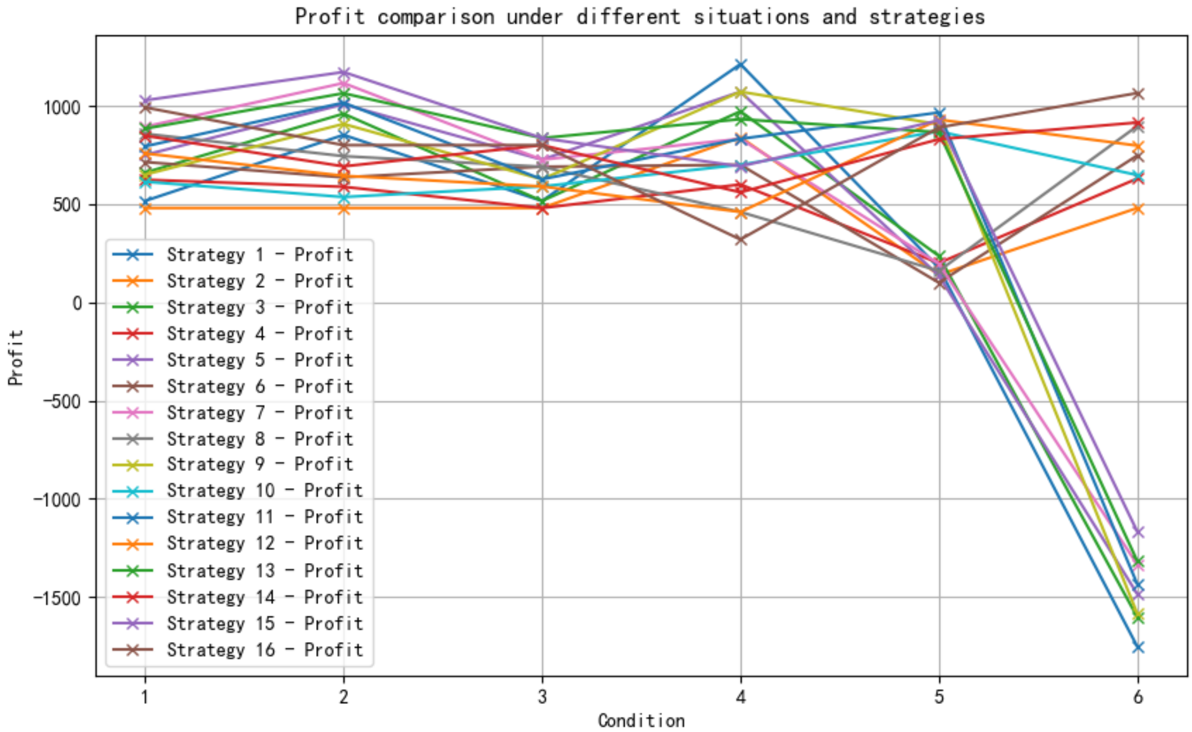

Figure 1. Comparison of profit and benefit under different decisions

As seen in Figure 1, for the six specific scenarios, each has a particular strategy that achieves the maximum profit value. However, each scenario's corresponding strategy varies due to differing costs, inspection, and disassembly conditions. For example, Strategy 15 is highly suitable for Situations 1 and 2, yielding the highest profits, but is not suitable for Situation 6, where profits are lower.

After evaluating the model and optimizing the dynamic strategies, this study ultimately identifies the optimal strategy for the six scenarios, as shown in Table 3.

Table 3. Optimal strategy scheme for six cases

Condition	Maximum profit(yuan)	Detect parts 1	Detect parts 2	Detect finished product	Dismantle nonconforming products
1	1308.00	No	No	No	Yes
2	1477.00	No	No	No	Yes
3	1104.00	No	No	Yes	Yes
4	1513.00	Yes	Yes	Yes	Yes
5	1244.00	No	Yes	No	Yes
6	1342.50	No	No	No	No

Through the analysis of the results, the impact of various factors on decision-making can be assessed. Comparing Situation 2 and Situation 3, this study finds that the defect rates of spare parts and finished products affect the maximum profit and the corresponding decisions, primarily reflected in the decision of whether to inspect the finished products. In comparing Situation 2 and Situation 4, this study observes that the inspection costs of spare parts and finished products also influence the maximum profit and the relevant decisions, particularly regarding whether to inspect the spare parts and whether to inspect the finished products.

4. Conclusion

For the sampling inspection model, this study classifies and discusses various scenarios based on different population sizes, addressing cases of small populations, limited capacities, and infinite capacities by establishing distinct distribution models. Through hypothesis testing, it determines the minimum sampling frequency that meets the confidence interval under both finite and infinite samples, ensuring the representativeness of the samples and enhancing the overall credibility of the products. The results indicate that this model significantly reduces inspection costs when the population size is large.

For the dynamic optimization model, this study first establishes a profit-effectiveness model for finished products based on process conditions, with the objective of maximizing benefits. Utilizing a dynamic optimization decision model, real-time adjustments are made based on the real-time data from the manufacturing process, allowing for better responses to fluctuations and changes that occur during production. Compared to static optimization, dynamic optimization provides immediate feedback and correction on actions such as whether to inspect processes, ensuring that the operations remain in an optimal state and meeting the practical needs of enterprises that frequently adjust process parameters in real production scenarios.

With the development of global economy and the progress of science and technology, production management is facing unprecedented challenges and opportunities. Data-driven production planning is one of the important trends in modern production management. By collecting and analyzing the data in the production process, enterprises can make production plans more accurately, achieve accurate decision-making and continuous improvement. In the future, the research will focus on the intelligent, green and data production plan to continuously improve the core competitiveness of the enterprise and achieve sustainable development.

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