

Optimization of Product Inspection and Disassembly Strategies Based on Cost-Benefit Analysis

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Abstract. In modern manufacturing, enterprises must develop optimal strategies by balancing product quality and cost management to maintain competitiveness and sustainable development in fierce market competition. This paper builds an optimal model for defect rate sampling inspection based on the central limit theorem and using the normal distribution to approximately fit the binomial distribution, as well as an optimization model for production inspection and disassembly decision-making based on hypothesis test theory and multi-standard decision analysis. With the help of the above two models, the enterprise obtained the optimal sample sizes for sampling and inspection in two situations: 180 and 126 pieces, respectively. The optimal production inspection and dismantling plans were analyzed for six cases with reasonable settings. The maximum expected profit per product for each case was 20.58 yuan/piece; 16.26 yuan/piece; 18.88 yuan/piece; 14.97 yuan/piece; 21.66 yuan/piece; 21.92 yuan/piece. According to the sample testing and disassembly plan obtained, enterprises can make better production plans to reduce production costs and get more profits.

Keywords: Hypothesis Testing, Multivariate Decision Models, Optimization Models, Center Limit Theorem, Normal Distribution

1. Introduction

In modern manufacturing, product quality control and cost management are key for enterprises to maintain competitiveness and achieve sustainable development [1-5]. With the intensification of market competition and the continuous improvement of consumers' requirements for product quality [6], the challenge faced by enterprises is not only how to improve the qualification rate of products and reduce the defective rate, but also how to reasonably control the inspectional cost, production cost, and after-sale service cost caused by defective products in the production process [7-8]. How to find a balance among these interdependent factors and develop optimal production and quality management strategies has become an important problem for enterprises to stand out in the market with intense competition [9-12].

In the actual production process, the additional production cost of finished products is mainly influenced by the acceptable defective rate of spare parts and the decision-making scheme of the production process. Therefore, this study decided to optimize from the above two aspects to achieve the lowest production cost. For the problem of the defective rate of production parts, this article sets two different situations to help enterprises determine whether to accept a certain batch of spare parts. At the same time, suitable sampling and inspection plans are designed for both situations, which can accurately help us determine whether to accept the batch of spare parts while achieving the lowest relative testing cost. For the design of production decision-making schemes, this article reasonably sets six cases based on actual research and data to simulate the different production situations faced by enterprises. Then, based on the set production situations, the three decisions that need to be considered are visually described. Due to the complexity of the situations involved and the fact that disassembly issues can be discussed separately during the research process, this article firstly designs a production decision plan that does not consider disassembly and calculates the optimal plan for each

Situation 1: Through hypothesis analysis, this is a one-sided test. So the study selects $\alpha=0.05$ and $\beta=0.10$. By checking the t-value table, it is found that:

$$Z_{0.05} = 1.645 \tag{4}$$

$$Z_{0.10} = 1.282 \tag{5}$$

Enter the corresponding data to solve for $n_1=180$.

Situation 2: Here the study takes $\alpha=0.10$ and $\beta=0.10$, according to the t-value table, the upper 10% critical value of the normal distribution at this time is:

$$Z_{0.10} = 1.282 \tag{6}$$

Thus, calculate and obtain $n_2=126$.

3. Optimization of Inspection and Disassembly Decisions in the Production Process

3.1. Actual Production Situation Setting and Decision-making Structure

In this study, the following factors were considered based on the actual production situation of the enterprise:

1. The finished product needs to be made up of multiple spare parts, and there may be a certain amount of defective products in spare parts and finished products during the production process. If there is any defective spare parts appear, the assembled product will be defective. But, even if all the spare parts are qualified, the assembled product may not be qualified. The defective rate conforms to the real production situation, which is neither too high nor too low (the defective rate of finished products refers to the ratio of defective products produced when all parts are qualified);

2. Enterprises can choose to inspect spare parts or finished products during the production process and discard the unqualified spare parts directly. Unqualified finished products can choose whether to dismantle them into spare parts and continue to be put into the production line (dismantling requires corresponding dismantling fees);

3. If the user purchases unqualified products, the enterprise will unconditionally exchange the products from the same production line (which may not necessarily guarantee qualification) and incur exchange losses (exchange losses refer to losses other than exchanging defective products, such as logistics costs, corporate reputation, etc.). The returned unqualified products can also be considered for disassembly;

4. The production line of the enterprise should be fixed, that means once it is decided whether each part and the finished product should be inspected and whether unqualified products are selected for disassembly or not, the inspection of each spare part and the finished product will not be changed after returning to the production line.

For such complex production management decisions, this paper reasonably set the following six cases as shown in Table 1:

Table 1. The Cases Encountered by Enterprises in Production

Cases	Spare part 1			Spare part 2			Finished product				Unqualified finished products	
	Defective rate	Unit price	Testing cost	defective rate	Unit price	Testing cost	Defective rate	Assembly Cost	Testing cost	Market price	Exchange losses	Dismantling cost
1	10%	4	2	10%	18	3	10%	6	3	56	6	5
2	20%	4	2	20%	18	3	20%	6	3	56	6	5
3	10%	4	2	10%	18	3	10%	6	3	56	30	5
4	20%	4	1	20%	18	1	20%	6	2	56	30	5
5	10%	4	8	20%	18	1	10%	6	2	56	10	5
6	5%	4	2	5%	18	3	5%	6	3	56	10	40

Note: The unit price, assembly cost, testing cost, exchange losses, market price, and dismantling cost are all in yuan per piece.

From Table 1, it can be seen that the finished product is only composed of spare parts 1 and 2, and the purchase unit price or assembly cost of the finished product is fixed at 4, 18, and 6, while the market price of the finished product is fixed at 56. For the setting of defect rate: in cases 1 and 3, all spare parts and finished products are set at 10%; Both cases 2 and 4 are 20%; The defective rate of spare part 1 and finished product in case 5 is 10%, and the defective rate of spare part 2 is 20%; All spare parts and finished products in case 6 are 5%. For the setting of testing costs: The testing costs for part 1 in six cases are 2, 2, 2, 1, 8, and 2, respectively; The testing costs for spare part 2 are 3, 3, 3, 1, 1, and 3, respectively; The inspectional costs in six cases are 3, 3, 3, 2, 2, and 3, respectively. For exchange losses, there are the exchange losses in six cases respectively: 6, 6, 30, 30, 10, and 10. For the dismantling cost of unqualified products, the first 5 cases are all 5, and case 6 is 40.

Figure 1 shows the complete process diagram of production process decision-making, which provides a clear understanding of the production, disassembly, inspection, exchange, and their interrelationships, as well as the decisions that need to be made.

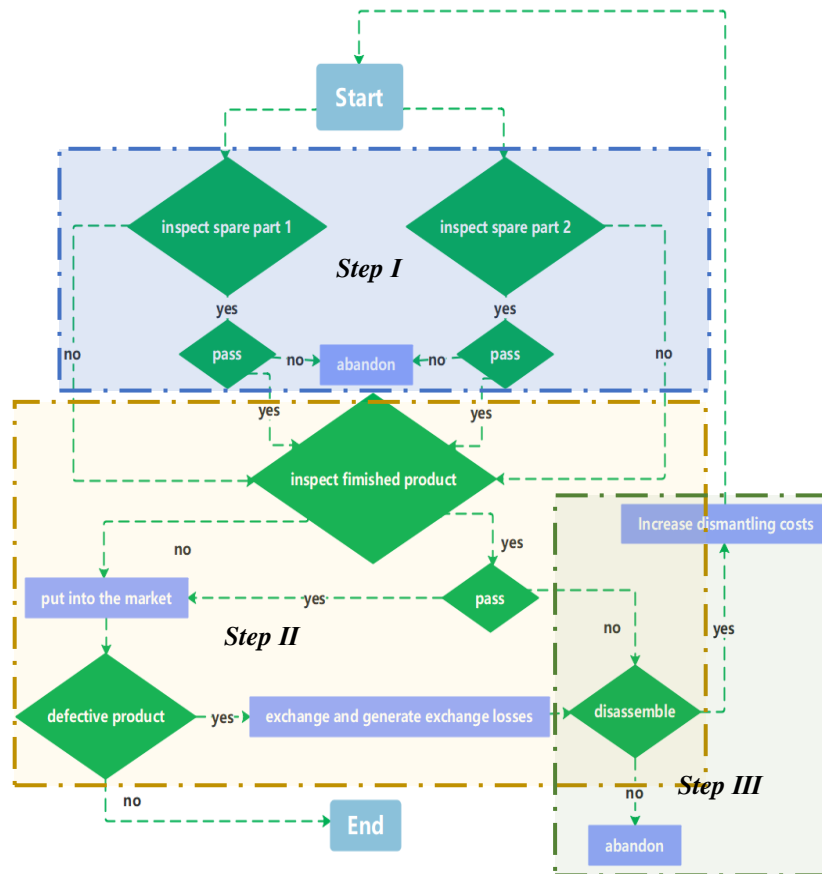


Figure 1. Decision Process Diagram

According to the analysis in Figure 1, it can be concluded that the decisions that need to be considered in this study mainly include:

- (1) Whether the spare parts 1 and 2 be inspected;
- (2) Whether the finished product be inspected;
- (3) Whether the unqualified finished products be disassembled.

According to the data in Table 1 and combined with actual situation analysis, the purchase unit price and the assembly cost of the finished product remain unchanged. The three are as follows:

$$x_1 = 4, x_2 = 18, x_3 = 6 \tag{7}$$

In addition, let the testing cost be y , the dismantling cost be z , the exchange loss be u , the defective rate be P , the market selling price be S , and the total production cost of all processes be M .

$$M_3 = (u + \frac{x_1 + J_a \cdot y_1 + x_2 + J_b \cdot y_2 + x_3}{P'_3})(1 - P'_3) \quad (13)$$

The profit model under various decisions, when the disassembly step is not considered, is:

$$W' = S - M_1 - M_2 - M_3 \quad (14)$$

3.3. Optimization Model for Inspection and Disassembly Production Decisions

1) The discussion of dismantling: the study records the value of dismantled qualified spare parts as M'_1 and records the loss of dismantling costs as z . Therefore, this paper only needs to compare the sizes of the above two, you can determine whether to dismantle them in different cases. According to the formula (2), it can be concluded that the dismantling of the net profit is:

$$M_c = M'_1 - z \quad (15)$$

When this value is greater than 0, choosing disassembly is better for enterprises; Conversely, leave the best plan as the last model.

2) Assuming that the unqualified finished product is disassembled and the obtained spare parts are re-run in steps (1) and (2). The question of whether to inspect them or not is discussed on a case-by-case basis since the inspecting of the spare part before the first assembly is different:

In a new round of production, the amount cost before assembly is set as M_a and the amount cost after assembly is set as M_b , without taking into account the costs incurred in the first round.

The paper set the new real finished product qualified rate as P''_3 : Considering the first assembly of finished products is already defective and taking into account the effect of the second inspection, the real defective rate of spare parts will change, resulting in the new finished product defective rate will also change. Due to the P'_3 has many situations, and the subsequent discussion will not involve all the situations, this paper below combines specific situations before calculating:

① Both 2 spare parts were inspected:

It can ensure that both are qualified spare parts, then step (1) does not need to re-inspect. For the finished product, it is necessary to compare the amount of inspection and neglect inspection, respectively. And the true qualified rate:

$$P''_3 = P_3^2 \quad (16)$$

Then at this point:

$$M_b = \begin{cases} \frac{x_3 + y_3}{1 - P''_3} & \text{Inspect the second finished product} \\ x_3 + (u + M_3)P''_3 & \text{Do not inspect the second finished product} \end{cases} \quad (17)$$

② One of the spare parts is inspected:

Decision-making discussion of step (1): it is known that the inspected spare part must be a qualified one when assembling and if the finished product is unqualified, then the probability of the two spare parts are both qualified is P_3 . And the uninspected spare parts of the true defective rate $1 - P_3$. This paper takes uninspected spare part 2 as an example of the calculation of M_a :

$$M_a = \begin{cases} \frac{y_2}{1 - P_3} & \text{Inspect the second finished product} \\ 0 & \text{Do not inspect the second finished product} \end{cases} \quad (18)$$

The case of spare part 1 is not inspected and is similar to the last case, just transforming the parameters and then respectively, combined with the specific case to calculate the minimum value of the respective $M_a + M_b$ and then compare.

Decision-making discussion of step (2): if the uninspected spare parts are inspected in the new cycle, then the situation after assembly will be the same as the first case; if they are not inspected, then when the two spare parts are assembled again, the probability of being able to get a qualified finished product is

$$P_3'' = (1 - P_3) \cdot P_3 \tag{19}$$

Combined with the actual situation, its maximum value is about 18% when P_3 is less than 25%. Considering the corporate image and product quality, the finished product must be inspected at this time.

③ Neither spare part has been inspected:

The probability that both two spare parts are qualified is P_3 , and the probability that two spare parts contain defective products is $1-P_3$. At this time, the two qualification rates are still in the unknown. The difference compared to the first round is that the real defective rates are elevated, which can be brought into the decision-making of the first round and analyzed in the context of the specific situation.

However, it is important to note that if it is decided to inspect only one spare part, then regardless of the purchase cost, the lower defective rate and inspection cost will be prioritized.

After considering disassembly, the final profit model is:

$$W = S - M_1 - M_2 - M_3 + W_c' \tag{20}$$

The meaning of W_c' : the net profit obtained by disassembling the spare parts, assembling them again, and selling them. Its optimal solution will be analyzed and solved in the model solution with the specific situation.

4. Result and Discussion

4.1. Sampling and Inspecting Program in Two Situations

Based on the solution results, this paper obtains the sample sizes for the two situations and designs a sampling and inspecting scheme as follows:

Situation one:

The paper randomly takes 180 spare parts from this batch for inspecting and statistics inspectional results. If the defective rate in the inspection results is higher than 10%, the batch will be rejected.

Situation two:

The paper randomly takes 126 spare parts from this batch for inspecting and statistics inspectional results. If the defective rate in the inspection results is less than 10%, the batch will be accepted.

4.2. Inspection and Disassembly Decisions Optimize Results for Each Case

4.2.1. Expected Profit on Individual Commodities when Dismantling is not Considered

Solving according to the model equation (13), Table 2 below shows the decision options and their expected profits when this expected profit is maximized:

Table 2. Decision table for pre-disassembly inspections

Cases	Whether to inspect spare parts 1	Whether to inspect spare parts 2	Whether to inspect the finished product	Expected profit on a single item before dismantling
1	no	no	yes	17.3877
2	yes	no	yes	6.9875
3	no	no	yes	14.9877
4	yes	yes	no	6.5000
5	no	no	yes	16.5432
6	no	no	no	21.9159

Note: In addition, a complete table of the different decision options and their expected profits in each case is shown in Appendix 1.

From the results shown in Table 2, the following conclusions can be drawn:

- 1) The decision making for each step before disassembly can be derived
- 2) When the defective rate is lower, choosing not to inspect the spare parts can obtain higher profits.
- 3) The quantitative relationship between inspectional cost and the purchase unit price has an impact on whether to inspect or not.

4.2.2. The Decision of whether to disassemble in each case

The conclusion of Table 2 will be brought into the formula (8), and this paper respectively determines whether to carry out disassembly. The study calculated case 1 as an example. The two spare parts qualified or not can be divided into four cases, their respective probabilities are as follows:

$$\left\{ \begin{array}{ll} P_3 = \frac{1}{10} & \text{All qualified} \\ (1 - P_3) \frac{P_1 P_2}{P_1 P_2 + (1 - P_1) P_2 + P_1 (1 - P_2)} = \frac{9}{190} & \text{All fail} \\ (1 - P_3) \frac{(1 - P_1) P_2}{P_1 P_2 + (1 - P_1) P_2 + P_1 (1 - P_2)} = \frac{81}{190} & \text{Only part 1 pass} \\ (1 - P_3) \frac{(1 - P_2) P_1}{P_1 P_2 + (1 - P_1) P_2 + P_1 (1 - P_2)} = \frac{81}{190} & \text{Only part 2 pass} \end{array} \right. \quad (21)$$

Excluding the inspectional cost, the actual price of purchasing a qualified spare part 1 and spare part 2 is ¥ $\frac{40}{9}$ per part and ¥20 per part respectively. Hence the value of the spare parts obtained after disassembly:

$$M'_1 = \left(\frac{40}{9} + 20\right) \times \frac{1}{10} + \frac{40}{9} \times \frac{81}{190} + 20 \times \frac{81}{190} \approx 12.86 > 5 \quad (22)$$

The net profit after dismantling is positive so dismantling is chosen.

From the above methodology, the dismantling decisions for the remaining cases can be obtained in the same way as in the table below:

Table 3. Decision table for dismantling

case	1	2	3	4	5	6
Disassemble or not	yes	yes	yes	yes	yes	no

As can be seen from Table 3 above, disassembly is required in all except case 6.

4.2.3. Decision-making options for post-disassembly inspection

Since case 6 chooses not to disassemble, only the decision options for the first five situations need to be analyzed in the following way:

1) The spare parts disassembled in cases 1, 3, and 5 are not inspected, but since the finished product is known to be defective, the true defective rate of the disassembled parts will be the initially given value. However, the rest of the cases are the same as if purchasing new spare parts for production. Therefore, the new defective rate can be brought into the first code for calculation and conclusions drawn.

2) In case 2, spare part 1 does not need to be inspected again. There are four decisions for spare part 2 and finished product inspection, of which the decision that both are not inspected ends up with a too low qualified rate of the finished product and too much of an impact on the company's image to be adopted. After calculating and comparing the remaining three decisions, the decision that inspects both is more profitable.

3) In case 4, spare parts are both inspected, and for the decision of inspecting the finished product, the calculation process is simpler and the comparison reveals that it is more profitable not to inspect the finished product.

As a result, this paper derives specific decisions when getting disassembled spare parts and repeating steps (1) and (2) as shown in Table 4:

Table 4. Decision table for post-disassembly inspection

case	Whether to inspect part 1	Whether to inspect part 2	Whether to inspect the finished product
1	yes	no	yes
2	no	yes	yes
3	yes	no	yes
4	no	no	no
5	no	no	yes

Tables 2, 3, and 4 provide a clear picture of the specific decision-making options throughout the process, which can be summarized as follows:

- 1) Metrics such as inspection costs, defective rates, exchange losses, and dismantling costs all affect the optimal decision scheme to varying degrees.
- 2) The necessity of finished product inspection is greater when exchange losses are excessive.
- 3) Disassembly is not performed when the cost of disassembly is too high that it is higher than the price of purchasing a pair of qualified spare parts. Care needs to be taken to improve the finished product.

The results of the corresponding indicators are shown in Table 5:

Table 5. Indicator Results

case	1	2	3	4	5	6
Expected profit without dismantling	17.3877	6.9875	14.9877	6.5	16.5432	21.9159
Ultimate expected profit	20.5777	16.2575	18.8823	14.972	21.6551	21.9159

The results of Table 5 were analyzed in this study as follows:

- 1) For the first five cases, After dismantling, restructuring, the revenue increased after reselling, indicating that dismantling is beneficial for profitability.
- 2) Case 6 has the highest final expected profit and its finished products are not disassembled to regain value, so it can be deduced that the defective rate has a greater impact on the cost of production.
- 3) Cases 2 and 4 end up with a bigger boost in expected profit than when they are not disassembled, with the common denominator being that at least one spare part is inspected at the first time, i.e. inspected spare parts are more valuable to disassemble.

5. Conclusions

For the optimal design of sampling inspection for defective rate, this study first determined a statistical model. The sampling inspection results followed a binomial distribution. Due to the large quantity required for enterprise production, it can be fitted as a normal distribution. The paper selects one-sided hypothesis testing based on the characteristics of this scenario and uses the Z-test to conduct hypothesis testing on normal variables. Conduct hypothesis tests on situation 1 and situation 2 respectively to determine their optimal sampling and testing plans: Situation 1 selects 180 spare parts for testing, and if the defective rate is higher than 10%, reject them; Situation 2 selects 126 spare parts for testing, and accept them if the defective rate is less than 10%. Finally, the model was validated to demonstrate its rationality and correctness.

For the selection problem of inspection and disassembly decisions in the production process, this study first set 6 reasonable cases based on actual conditions, and visually analyzed the production decision-making process. Then, according to the different cases the paper combined with reasonable data constraint preprocessing to simplify the model. Then the study divided the production process into four stages: pre-assembly, post-assembly to pre-sale, post-sale to pre-disassembly, and entering the cycle after disassembly. Firstly, the study established the optimal solution decision model and its profit for the first three stages without considering disassembly. Then, the study established the decision model for disassembly. In the case of the disassembly decision, further establish the decision

model for the spare parts obtained from disassembly entering a new round of production, which can maximize the profit after disassembly. The optimal solutions for the six cases were ultimately obtained, with the maximum expected profit respectively for each product: 20.58 yuan/piece; 16.26 yuan/piece; 18.88 yuan/piece; 14.97 yuan/piece; 21.66 yuan/piece; 21.92 yuan/piece.

This paper mainly provides a research approach and framework for the optimal decision optimization of various links in the production process of quality control and management-related fields. The study establishes an appropriate mathematical model, which is applicable to different actual production situations of enterprises and proves the feasibility and correctness of the method. It has certain practical reference value for decision-making schemes in the actual production of enterprises.

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