

Decision-making in the production chain of a firm based on a dynamic planning model with backward recursion

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Abstract. The inspection of spare parts and finished products in the current production process is often difficult to adapt to the requirements of a variety of production scenarios, in order to meet the minimization of production costs in a variety of production scenarios, this paper takes six production scenarios as an example, in order to achieve the optimization goal of minimizing the cost of enterprise production, the use of the dynamic programming model for the four phases of the enterprise's production of the inverse recursive solution for enterprise production. This paper explores the necessity of testing both individual components and finished products, as well as the practicality of dismantling nonconforming products at various stages of the production process under different conditions. It further examines whether defective products should undergo disassembly at each phase of production and assesses if testing components and final outputs, along with handling defects through disassembly, should be conducted differently depending on specific circumstances.

Keywords: dynamic production decision-making, uncertainty management, dynamic planning models.

1. Introduction

Enterprises often need to make a variety of decisions in the production process to minimize costs and improve profitability, however, the production of enterprises is a complex process that contains a variety of production scenarios with dynamic changes, in the research on enterprise production decision-making in a variety of production scenarios, the current existing theoretical and empirical results still have limitations that make them unable to satisfy the enterprise's decision-making needs in a complex environment. In research on corporate production decision-making, Davide et al.^[1](2021) and Anna et al.^[2] (2021) suggest that many current models of production decision-making are highly dependent on complex mathematical calculations. Despite the theoretical value of these models, the complex mathematical reasoning makes it difficult for business managers to understand and apply them in practice, which in turn limits the generalizability of these models in dynamic production environments. This makes it difficult for firms to make effective decisions quickly in the face of changing production situations. MacDonald et al.^[3] (2022) and Ahmed et al.^[4](2020) point out that firms need to consider the coordinated relationship between multiple stakeholders when making production decisions, and this coordination is crucial in the implementation of quality inspection and production strategies. However, existing studies still do not discuss this critical aspect in sufficient depth, making it difficult for firms to achieve comprehensive and efficient decision-making in dynamic production environments.

Further, Demirel et al.^[5] (2019) and Yu et al.^[6] (2019) argue that the production process of firms is full of dynamically changing factors such as fluctuations in production demand and supply chain instability, but existing studies tend to assume that production environments are static and fail to adequately consider the impacts of these dynamics on decision making, which makes it difficult for companies to make flexible adjustments in the face of rapidly changing markets. Therefore, it is particularly important to dynamically adjust the inspection strategy for the inspection of defective rate of spare parts and finished products. Teece^[7] (2018), Bocken and Geradts^[8] (2019) proposed that dynamic planning models can effectively deal with the production decision problem of enterprises under the uncertain conditions of demand fluctuation and supply chain

disruption. They pointed out that by combining scenario analysis, robust optimization, and demand response, dynamic planning models can optimize facility siting, inventory management, and resource allocation, thus enhancing the decision-making efficiency and stability of enterprises. The issue of data quality and access should also not be neglected in model selection, Levenson and Fink^[9](2017) and Talesh^[10] (2019) emphasized that many enterprises often face the problem of incomplete or inaccurate data in practice, and these data deficiencies limit the practical application of decision-making models based on historical data, making it difficult to adapt to rapidly changing production conditions. Therefore, they argue that dynamic programming models have advantages in dealing with data deficiencies, but further exploration is needed to apply them to multiple contexts of spare parts and finished product inspection in specific production processes.

Therefore, our study fills the gap in the existing literature on dynamic decision-making models. In response to many studies assuming that the production environment is static, we consider factors such as demand fluctuations, supply chain changes, and production condition dynamics, and further focus on multi-stakeholder coordination issues by analyzing the substandard rate of parts and finished product inspection, balancing the needs and goals of different departments in production decisions, and providing a set of effective quality inspection metrics. Based on this, a more adaptable decision-making framework is developed through the use of dynamic programming models to support more accurate decision-making in complex production environments. This framework allows companies to flexibly adjust their inspection strategies in rapidly changing market environments to effectively respond to the challenges of different scenarios and meets their requirements for dynamic analysis. By deeply analyzing the inspection defect rates of parts and finished products, our study can provide more accurate decision support, which will help companies make more effective production decisions in a variety of production scenarios, and enhance their competitiveness and flexibility in a rapidly changing market environment.

2. Relevant theories

Dynamic Programming (DP) is an optimization technique for solving complex decision problems, especially those scenarios that can be decomposed into smaller, overlapping subproblems. Its core idea is to decompose a large problem S into multiple smaller subproblems s , and construct the overall optimal solution by solving these subproblems. It is usually solved recursively. $V(S)_{max}$ which is usually solved recursively, can significantly reduce the computational complexity and avoid repeated computations, thus providing an efficient solution when dealing with large amounts of data and variable conditions.

Let's set up a large problem S can be decomposed into several subproblems s that dynamic programming transforms the problem into solving the optimal solution to a set of recursive equations. If $V(s)$ represents the optimal solution under the subproblem s the optimal solution under the subproblem, then the goal of dynamic programming is to obtain the optimal solution of the overall problem by combining the optimal solutions of all the subproblems $V(S)$. Suppose that $V(S)$ Denote the:

$$V(s) = \max\{C(s) + \sum_{i=1}^n V(s_i)\} \quad (1)$$

Among them, $C(s)$ is the benefit or cost of the decision in the state, the benefit or cost of the state, and s_i denotes the benefit or cost of the decision in the associated subproblems. By such recursive computation, dynamic programming is able to find the globally optimal solution while avoiding repeated computations.

In the optimization of enterprise resource allocation, the total amount of resources can be set to be R , and the marginal benefit from each resource unit is b_i . Assume that each decision option D_i requires a resource of r_i and its benefit is p_i , then the firm's objective is to choose the combination

of options that maximizes total return within the total resource R constraints, to select the combination of options that maximizes the total return. This is expressed in terms of the dynamic programming formula:

$$\max \sum_{i=1}^n (p_i + b_i \cdot r_i) \times D_i \quad (2)$$

The constraints are:

$$\sum_{i=1}^n r_i \times D_i \leq R \quad (3)$$

Through this formula, the optimal solution of each sub-problem can be solved step by step under the framework of dynamic programming, combining the enterprise's demand for resource allocation with the expectation of revenue, so as to construct the optimal scheme of overall re-source allocation. At the same time, future demand fluctuations are predicted through probabilistic modeling, such as assuming that the demand obeys a normal distribution $N(\mu, \sigma^2)$. In this way, the resource allocation strategy can be adjusted under the uncertainty conditions, and the adaptability and competitiveness of the enterprise in the complex market environment can be improved in a dynamic optimization way.

3. Experimentation

Quality inspection and resource optimization are important aspects of a company's production process that determine economic efficiency. The complexity and dynamics of the production process often make it challenging for companies to decide whether or not to test spare parts and finished products, and how to deal with non-conforming products in order to minimize production costs. In making these decisions, companies not only need to consider direct economic factors such as inspection and disassembly costs, but also need to cope with the uncertainty caused by dynamic environments such as fluctuations in defect rates, changes in market demand, and instability in the supply chain. In order to explore the enterprise in the production process for the inspection of spare parts and finished products for decision-making, we simulated the enterprise's real production process, for the production of a variety of problems that may be encountered in the production process of this paper on the enterprise production process for the following assumptions:

Two types of spare parts and finished product defect rates are known.

First, whether the spare parts (part 1 and/or part 2) are tested or not, if a certain spare part is not tested, this spare part will go directly to the assembly; otherwise, the detected unqualified spare parts will be discarded. Subsequently, whether to test each piece of assembled finished products, if not tested, the assembled finished products directly into the market; otherwise, only qualified finished products enter the market. Next, whether the detected unqualified finished product is disassembled, if not disassembled, the unqualified finished product is directly discarded; otherwise, for the disassembled spare parts, step one and step two are repeated. Finally, for the unqualified products purchased by the user, the enterprise will unconditionally exchange them and incur a certain exchange loss (such as logistics costs, enterprise reputation, etc.). Repeat step 4 for returned nonconforming products.

This involves a number of decision-making aspects of the production process of the enterprise. Among other things, the goal is to maximize the combined profit generated by the enterprise during the production, testing and dismantling processes, while meeting the quality requirements of production.

Create an objective function where net profit can be obtained by subtracting total costs from total revenue:

$$P = R - C \quad (4)$$

Where P stands for Net Profit, R stands for Total Revenue and C stands for Total Cost.

Based on the assumption that the information is known to be easily accessible to the total revenue, this is therefore essentially for solving the minimization of the total cost of the production decision process.

Total costs include parts purchase costs, assembly costs, inspection costs, exchange losses, and dismantling costs. Total revenue is based on the number of finished products sold and takes into account the effect of the defective rate.

The decision variables are whether to test or not to test spare parts and finished products, and whether to dismantle substandard finished products.

3.1. Disassembly Decision Modeling Establishment

Enterprises in the production often will not directly discard the defective products, but according to the specific circumstances of the parts for selective disassembly, so in the finished product testing process and exchange to retrieve the process of the defective products can choose to discard or dismantling to retrieve the process, which is in line with the actual production process of the enterprise, after dismantling the spare parts will not be damaged, can be reintroduced into the production, but the dismantling itself requires a certain amount of expenses, incurring a certain amount of cost, the cost of dismantling is not only the cost of production, but also the cost of production. Therefore, this paper analyzes the dismantling decision-making model by establishing it:

When the cost of dismantling is lower than the revenue generated by recycling spare parts, dismantling is carried out, otherwise it is discarded.

In this question, the formula for dismantling costs is

$$C_{dis} = dis_cost \times p_{prod} \times num_roducts \quad (5)$$

Among them.

dis_cost: Cost of dismantling each non-conforming finished product.

p_{prod} : The defective rate of the finished product, i.e. the proportion of the finished product that is substandard.

num_roducts: The number of finished products produced.

The formula for calculating the recovery benefit is

$$V_{rec} = sale_price \times p_{prod} \times num_roducts \quad (6)$$

Among them.

sale_price: The selling price of the finished product.

If $V_{rec} > C_{dis}$, then disassembly is recommended; otherwise, discard.

3.2. Construction of recursive equations

After clarifying the decision basis for disassembling and discarding the non-conforming finished products, we further integrate this strategy into the overall framework of dynamic planning, and realize the organic connection between various production stages in the form of recursive model. Through the cost-benefit analysis of the disassembly process, we can determine the optimal treatment of nonconforming products, which directly affects the inspection strategy and production arrangement in the preceding stages. In the construction of the recursive model, we take the disassembly decision as a key link, and through the backward and forward backward push, the state transfer of each stage is closely integrated with the optimal solution calculation, forming a complete dynamic decision path covering spare parts inspection, finished product inspection and processing, which provides a clear model logic and optimization direction for the experiment. As shown in Figure 1.

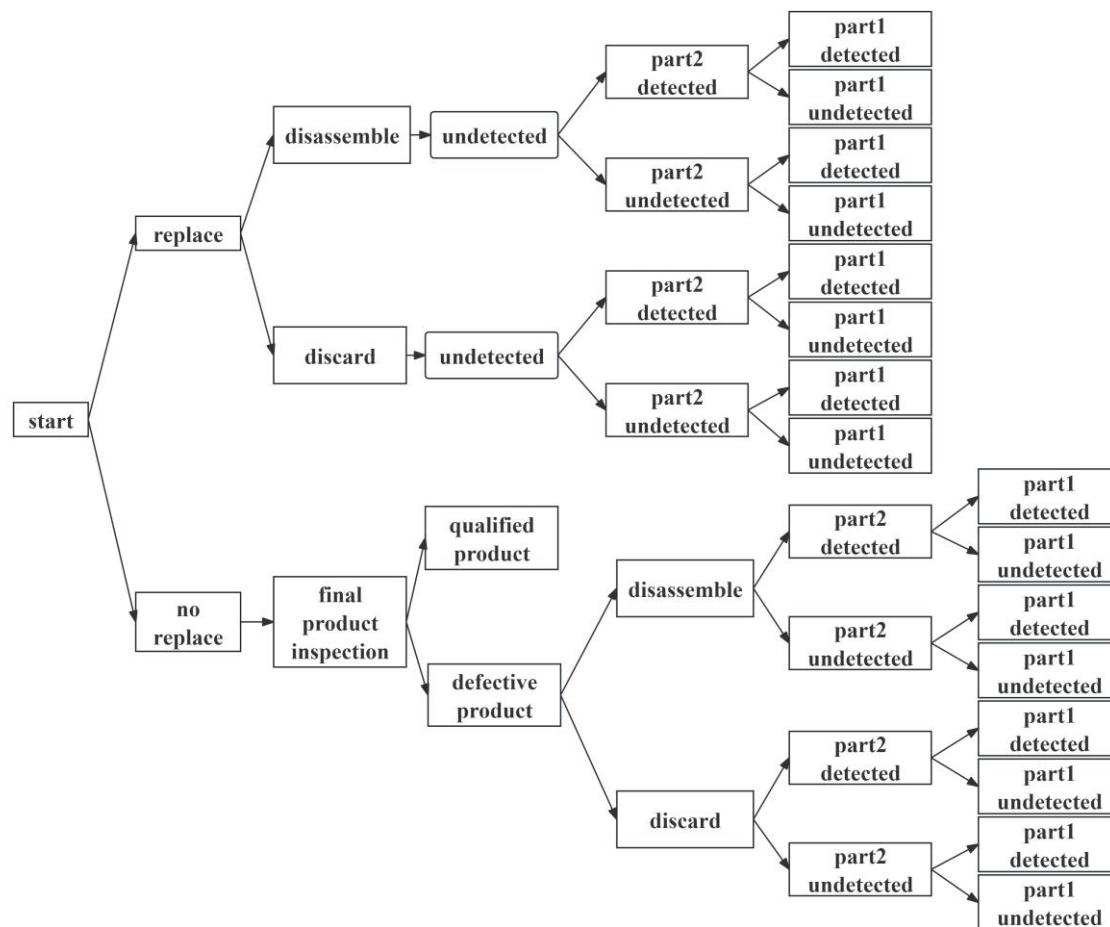


Fig. 1 Backtracking solution process for dynamic programming

This recursive method shows the dependency relationship between each link through a clear logical hierarchy, which makes it easy for enterprises to simulate the effect of strategies in different situations in actual production, quickly find the optimal production management plan, effectively reduce the computational complexity, reuse the intermediate results by using the dynamic planning idea, and flexibly cope with changes in the dynamic environment such as fluctuations in the rate of substandard products or adjustment of the inspection cost.

The stages of dynamic programming are categorized according to the figure above, which decomposes the complex multi-stage problem into a series of sub-problems, each representing a stage of decision making. Stages 1-4, are whether to test part 1, whether to test part 2, whether to test the finished product, and how to dispose of the nonconforming finished product (disassemble or discard).

And then define the state, in this question, we can define the following variables: S_{1-4} , respectively, are whether to detect part 1, whether to detect part 2, whether to detect the finished product, and throw the unqualified finished product or disassemble the unqualified finished product, and the state values are $\{0,1\}$. Among S_4 In this case, discard is 1 and disassemble is 0.

Subsequently, based on the above information, the state transfer equations are constructed and derived from the backward direction. If derived from the forward direction, each state will involve more possible paths, leading to redundant computation. In contrast, backward derivation can effectively limit the size of the computation and avoid repetition by targeting only the optimal results of the subsequent states. At the same time, each stage has explicit state transfer equations whose result values depend on more subsequent states. Backpropagation starts from the final stage and can resolve these dependencies layer by layer.

Stage 4: Disposal of non-conforming finished products, including both disassembly and disposal options

If you choose to **dismantle** the non-conforming finished product, the cost is:

$$Cost(S_4 = 1) = p_{prod} \times num_{product} \times disassemble_{cost} \quad (7)$$

Of which $disassemble_{cost}$ is the cost of dismantling nonconforming finished products.
If you choose to **discard** the nonconforming finished product, the cost is:

$$Cost(S_4 = 0) = 0 \quad (8)$$

(No cost is incurred for discarding nonconforming finished goods).
State transfer equations:

$$Cost(S_4) = \min(Cost(S_4 = 1), Cost(S_4 = 0)) \quad (9)$$

Stage 3: Finished product inspection

If you choose to test the finished product, the cost is:

$$Cost(S_3 = 1) = det_{prod} \times num_{product} + Cost(S_4) \quad (10)$$

Among them det_{prod} is the cost of testing the finished product.

If the finished product is not tested, the cost is:

$$Cost(S_3 = 0) = p_{prod} \times L_{return} \times num_{product} + Cost(S_4) \quad (11)$$

where p_{prod} is the defective rate of the finished product.

State transfer equations:

$$Cost(S_3) = \min(Cost(S_3 = 1), Cost(S_3 = 0)) \quad (12)$$

Stage 2: Part 2 Inspection

If Inspection Part 2 is selected:

$$Cost(S_2 = 1) = det_2 \times num_{product} + Cost(S_3) \quad (13)$$

where det_2 is the cost of testing part 2.

If part 2 is not tested:

$$Cost(S_2 = 0) = p_2 \times L_{return} \times num_{product} + Cost(S_3) \quad (14)$$

where p_2 is the defective rate of part 2.

State transfer equations:

$$Cost(S_2) = \min(Cost(S_2 = 1), Cost(S_2 = 0)) \quad (15)$$

Stage 1: Part 1 Inspection

If you choose to detect part 1:

$$Cost(S_1 = 1) = det_1 \times num_{product} + Cost(S_2) \quad (16)$$

where det_1 is the cost of testing part 1.

If part 1 is not tested:

$$Cost(S_1 = 0) = p_1 \times L_{return} \times num_{product} + Cost(S_2) \quad (17)$$

where p_1 is the defective rate of part 1, and L_{return} is the replacement loss.

State transfer equations:

$$Cost(S_1) = \min(Cost(S_1 = 1), Cost(S_1 = 0)) \quad (18)$$

Through the above modeling process, we simulate the decision-making process of the enterprise in the production process, list the costs of each stage in the production of the enterprise through the method of reverse recursion, and the assumption of the four steps of the cascade covers the whole process from spare parts to finished products to after-sale links, which takes into account the balance of cost-benefit in the dynamic production environment, which is logically complete and has a

practical basis. Therefore, experimentally verifying the cost-benefit relationship in different contexts can further illustrate the adaptability and optimization value of the hypothesis.

4. Results

According to the model established above, this paper obtains the minimum total cost, i.e., the optimal decision-making scheme, by substituting the specific parameter values of the six scenarios of the enterprise's production, and writing a program in python to perform the calculations, and continuously recursively.

The final results are shown in Table 1:

Table 1. Decision-making scenarios by scenario

	Situation 1	Situation 2	Situation 3	Situation 4	Situation 5	Situation 6
Part 1 Inspection	F	F	T	T	F	F
Part 2 Inspection	F	F	F	T	T	F
Finished Product Inspection	F	F	F	T	F	F
Defective Products' Disposal	T	T	T	T	T	F

From the table, we can analyze the cost, revenue, net profit and decision paths in detail for different scenarios, and the specific calculated values for different scenarios are shown in Table 2:

Table 2. Calculation results

Situation	Maximum Net Profit (Yuan)	Total Cost (Yuan)	Total Revenue (Yuan)	Part 1 Inspection	Part 2 Inspection	Finished Product Inspection	Defective Products' Disposal
Situation 1	20.10	30.30	50.40	F	F	F	T
Situation 2	12.20	32.60	44.80	F	F	F	T
Situation 3	13.90	36.50	50.40	T	F	F	T
Situation 4	11.80	33.00	44.80	T	T	T	T
Situation 5	18.90	31.50	50.40	F	T	F	T
Situation 6	23.70	29.50	53.20	F	F	F	F

As can be seen from the data analysis in Table 2, the net profit in case 6 reaches its maximum value. It shows that when choosing not to carry out several tests and treatments in the production process of the enterprise, the rate of defective products in the finished product has a relatively small negative impact on the final profit due to the low cost associated with it. Therefore, firms can achieve higher levels of net profit by adopting strategies that reduce operational costs.

Further observation reveals that as the number of inspections continues to increase, the enterprise's net profit shows a gradual decline. In particular, in case 4, when the enterprise carries out comprehensive testing and recycling, although the defective rate is effectively reduced, at the same time, the related cost increases significantly, which leads to the lowest point of the enterprise's net profit.

Overall, when inspection and disposal costs are relatively high and the defective rate has been maintained at a low level, firms tend to achieve greater net profitability by choosing the strategy of not performing additional inspections.

5. Summary

In this paper, we study the decision-making problem under various production scenarios in the production process of an enterprise, focusing on the inspection of spare parts and finished products and the treatment of nonconforming products to make an optimization analysis. The decision paths are optimized to minimize the production cost by using a dynamic programming model in the reverse direction of the production process. The results show that different inspection and dismantling strategies will affect the total cost and profit of the enterprise, and each inter-connected production link will affect the economic efficiency of the whole enterprise production, which needs to be considered by the enterprise in the whole production link.

Meanwhile, the findings of this paper verify the reliability of the dynamic planning model in complex production decision-making. By reasonably adjusting the detection and processing strategies, enterprises can improve the flexibility and accuracy of decision-making in the dynamically changing market environment. Future work can further extend the model and apply it to more complex production scenarios to help enterprises optimize production decisions in the face of uncertain market demand and supply chain fluctuations. And with the global emphasis on sustainable production and environmental protection, future research can consider more social and environmental factors in the model. By balancing economic efficiency and environmental protection in decision making, companies can not only reduce production costs, but also comply with the requirements of social responsibility and green production, which can further improve their social image and long-term competitiveness.

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