# Crop Planting Planning Based on the Branch and Bound Method

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**Abstract.** With the increasing scarcity of arable land resources in rural China, enhancing agricultural production efficiency on limited land has become a key objective in the development of modern agriculture. Existing studies have widely applied system dynamics, multi-objective mathematical programming, and other methods in the allocation of agricultural resources, but optimization strategies tailored to the rural environment in China are still relatively lacking. Taking a rural area in North China as an example, this paper establishes a mixed integer optimization model based on the actual planting data and constraints of the area, aiming to maximize the planting profit from 2024 to 2030. The branch and bound method is used to solve two scenarios: one where the excess production is discarded, and the other where it is sold at a discount, resulting in maximum profits of 38.5 million yuan and 57.37 million yuan, respectively. The results show that the constructed model effectively optimizes the planting profit, verifying the potential application of the branch and bound method in complex planting planning, and providing a scientific planting strategy for rural areas.

**Keywords:** Planting Profit, Mixed Integer Optimization, Branch and Bound Algorithm.

## 1. Introduction

As arable land resources in rural areas of China become increasingly scarce, how to achieve efficient agricultural production on limited land resources has become a key task in the development of modern agriculture. Rural arable land is often fragmented and scattered, with complex terrains, which increases the difficulty of formulating scientific planting strategies. The rational use of arable land resources and the optimization of planting structures are not only directly related to the increase in agricultural output and economic benefits but also of great importance in controlling planting risks and ensuring the sustainability of production.

In recent years, optimization studies on crop planting have employed methods such as system dynamics, multi-objective mathematical programming, constraint programming, and emergy evaluation to improve resource allocation efficiency and sustainable development. Barati et al. [1] used Vensim PRO and multi-objective mathematical programming to optimize crop patterns in the Kermanshah Plain of Iran, achieving a balance between income and water resources. Najafabadi et al. [2] optimized crop planting in Isfahan Province through multi-objective planning to save water and increase income. Mellaku and Sebsibe [3] reviewed literature, emphasizing the potential of model decision-making for agricultural resource efficiency in developing countries. Adamo et al. [4] optimized intercropping systems with constraint programming, reducing the use of pesticides and water. Yang et al. [5] demonstrated the balance of economic and ecological benefits with their emergy evaluation model. Alabdulkader et al. [6] optimized Saudi Arabia's cropping pattern, boosting net returns and saving 53% water and 48% land, focusing on water-efficient crops. Juwono et al. [7] optimized cropping patterns in Parsanga, Madura Island, using linear programming, improving water use and production compared to the existing paddy-paddy-second crop rotation. Wang et al. [8] employed the non-dominated sorting genetic algorithm (NSGA-II) to optimize the planting structure in the upstream region of Baiyangdian Lake, focusing on enhancing ecological sustainability while addressing regional agricultural production efficiency. Although these studies have shown the broad application potential of mathematical modeling in the efficient use of agricultural resources and sustainable development, research on planting strategies specific to the rural environment in China is still relatively lacking.

This paper, based on the specific situation of a rural area in North China, constructs a mixed-integer optimization model using the branch and bound method, aiming to maximize planting profits from 2024 to 2030 while meeting various constraints such as planting diversity and plot area restrictions. This is the first time the branch and bound method has been applied to such rural planting strategy optimization problems, providing a new theoretical basis and decision support for agricultural planting strategies in the region.

The structure of the paper is as follows: The first part includes the literature review and the reason to do this research. The second part discusses relevant theories, introducing the branch and bound method and its application in optimization problems; The third part is about model construction and experimental methods, elaborating on the establishment of the objective function and constraints; The fourth part presents experimental results and analysis, showing the solution results of the model under different conditions and optimization strategies; Finally, the fifth part summarizes the research conclusions and proposes future work prospects.

## 2. Relevant Theories

Considering that this is a mixed-integer programming problem, this paper adopts the branch and bound method [9] as the solution strategy. The core idea of the branch and bound method is to gradually divide the entire feasible solution space into smaller subsets, forming what is known as "branches." Within each subset generated by the branches, a lower bound value of the objective function is calculated for the solution space (for minimization problems), which is referred to as "bounding." After each branching operation, if the lower bound of a subset's objective function has already exceeded the objective value of the currently known feasible solution, then that subset will no longer be further branched, effectively reducing the search space and avoiding unnecessary calculations.

This process is iteratively repeated, with branching and bounding alternating until a feasible solution is found. Ultimately, the objective value of this feasible solution will not be greater than the lower bounds of any subset, thus ensuring the discovery of the global optimal solution. Therefore, the branch and bound method is an effective global optimization approach that can typically yield the optimal solution to the problem. The specific solution process is shown in Figure 1 [10], which demonstrates the complete process from branching to bounding:

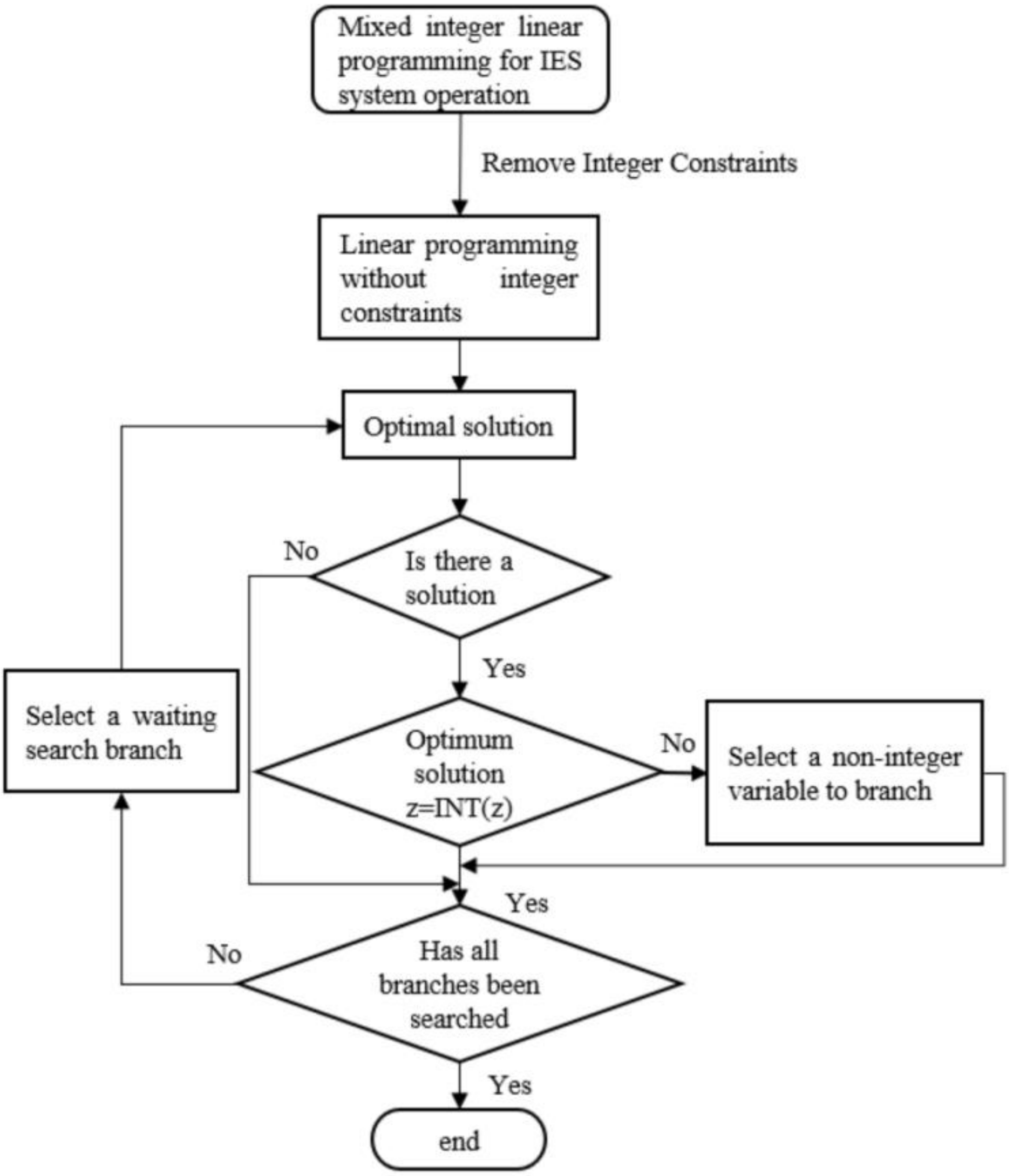


**Figure 1.** Flowchart of the Branch and Bound Method

## 3. Experiments

The data presented in this article is sourced from the official webpage of the National University Student Mathematical Modeling Competition, which can be accessed through the URL: https://www.mcm.edu.cn/html_cn/node/a0c1fb5c31d43551f08cd8ad16870444.html. This paper discusses the unsold vegetables in two scenarios. The first scenario involves directly discarding the vegetables, while the second scenario sells them at a 50% discount of the sales price. In order to obtain the maximum total revenue from the sale of crops over seven years for each scenario, this paper first sets integer variables $y_{ijt}$ to represent the planting area of each crop in each year, formulates the objective functions for both scenarios, determines the constraints, and finally uses the branch and bound method to obtain the results. The following figure 2 explains the overall experimental approach of this paper:

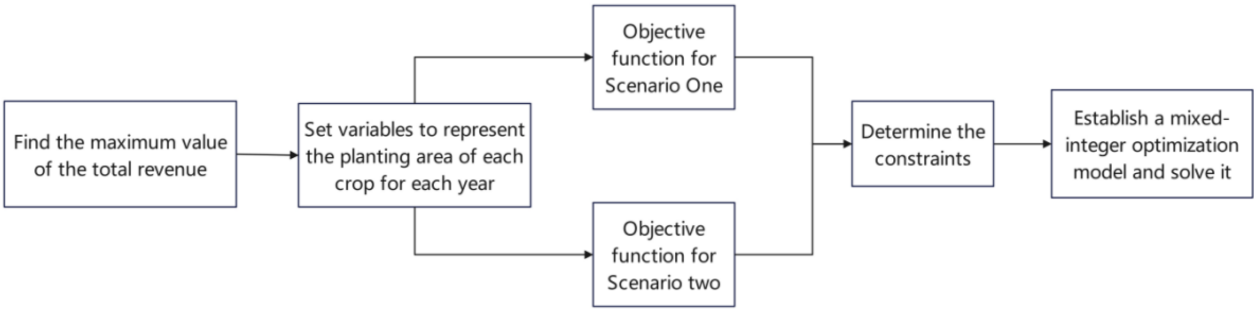


**Figure 2.** Flowchart of the Experimental Approach

### 3.1. Establishing the Objective Function

In order to simplify the model and ensure that the planting area of each crop in each season should not be too scattered, and the planting area of each crop in a single plot (including greenhouses) should not be too small, this paper divides each plot into two half zones. Following Adamo T, et al. [4], integer variable $y_{ijt}$ is adopted to indicate that several half zones were planted in the t year of the j plot of the i crop, that is:

$$y_{ijt}=\begin{cases}0, \text{Should not plant this type of crop}\\ 1, \text{Plant this type of crop on } 50\% \text{ of the area}\\ 2, \text{Plant this type of crop on } 100\% \text{ of the area}\end{cases} \tag{1}$$

Assuming that the expected sales volume for subsequent periods is equivalent to the total yield in 2023 $D_i$ , the calculation formula for the total production of crops in the current season is:

$$\sum_{j=1}^{78}\frac{A_j}{2}\cdot y_{ijt}\cdot q_{ij}=Q_{it}^{'} \tag{2}$$

Where $A_j$ is the planting area of the *j*-th plot, $q_{ij}$ is the yield per acre of *i*-th crop on the *j*-th plot, and $Q_{it}'$ is the total production of the *i*-th crop in the current season for the *t*-th year.

Similarly, the calculation formula for the total production of off-season crops is:

$$\sum_{j=79}^{82}\frac{A_j}{2}\cdot y_{ijt}\cdot q_{ijt}=Q_{it}^{''} \tag{3}$$

where $Q_{it}''$ represents the total production of the *i*-th off-season crop in the *t*-th year.

**(1) Scenario 1:**

When the total production exceeds the expected sales volume and the excess cannot be sold normally, the objective function is obtained as:

$$\begin{aligned}\max \sum_{t=2024}^{2030}\sum_{i=1}^{41}&\left((\min(Q_{it}^{'},D_i^{'})\cdot P_i)-\sum_{j=1}^{78}(C_{ij}\cdot\frac{A_j}{2}\cdot y_{ijt})\right)\\ +\sum_{t=2024}^{2030}\sum_{i=1}^{41}&\left((\min(Q_{it}^{''},D_i^{''})\cdot P_i)-\sum_{j=79}^{82}(C_{ij}\cdot\frac{A_j}{2}\cdot y_{ijt})\right)\end{aligned} \tag{4}$$

where $D_i'$ is the expected sales volume of the *i*-th current-season crop, $D_i''$ is the expected sales volume of the *i*-th off-season crop, $P_i$ is the sales price per unit of the *i*-th crop, and $C_{ij}$ is the planting cost of the *i*-th crop on the *j*-th plot.

**(2) Scenario 2:**

When the total production exceeds the expected sales volume and the excess will be sold at a 50% discount of the original 2023 sales price, the objective function is obtained as:

$$\max \sum_{t=2024}^{2030}\sum_{i=1}^{41}\left((\min(Q_{it}^{'},D_{i}^{'})\cdot P_{i})-\sum_{j=1}^{78}(C_{ij}\cdot\frac{A_{j}}{2}\cdot y_{ijt})+\max(0,Q_{it}^{'}-D_{i}^{'})\cdot P_{i}\cdot 50\%\right)$$
$$+\sum_{t=2024}^{2030}\sum_{i=1}^{41}\left((\min(Q_{it}^{''},D_{i}^{''})\cdot P_{i})-\sum_{j=79}^{82}(C_{ij}\cdot\frac{A_{j}}{2}\cdot y_{ijt})+\max(0,Q_{it}^{''}-D_{i}^{''})\cdot P_{i}\cdot 50\%\right) \quad (5)$$

### 3.2. Constraints

Flat dry land, terraced fields, and sloping land can only be used for single-season cultivation of food crops (except for rice), that is:

$$\sum_{i=16}^{41} y_{ijt}=0,\ j=1,2,\cdots,26,\forall t \quad (6)$$

Irrigated land can be used for planting rice or vegetable crops in the first season (single-season rice cultivation is counted as rice cultivation in the first season), that is:

$$\sum_{i=1}^{15} y_{ijt}+\sum_{i=35}^{41} y_{ijt}=0,\ j=27,28,\cdots 34,\forall t \quad (7)$$

In the second season, irrigated land can only be used for planting Chinese cabbage, white radish, and red radish, that is:

$$\sum_{i=1}^{34} y_{ijt}+\sum_{i=38}^{41} y_{ijt}=0,\ j=55,56,\cdots 62,\forall t \quad (8)$$

Ordinary greenhouses can only grow vegetable crops (excluding Chinese cabbage, white radish, and red radish) in the first season, and smart greenhouses can only grow vegetable crops (excluding Chinese cabbage, white radish, and red radish) in both seasons, that is:

$$\sum_{i=1}^{16} y_{ijt}+\sum_{i=35}^{41} y_{ijt}=0,\ j\in[35,54]\bigcup[79,82],\forall t \quad (9)$$

In the second season, ordinary greenhouses can only be used for cultivating edible mushrooms, that is:

$$\sum_{i=1}^{37} y_{ijt}=0,\ j=63,\cdots,78,\forall t \quad (10)$$

For flat dry land, terraced fields, and sloping land, the same crop cannot be planted for two consecutive years (for the $y_{ijt}$ that are 1 for two consecutive years, it is considered as being planted in two different areas of the same piece of land for two years, which does not violate the constraint), that is:

$$y_{ijt}+y_{ij(t+1)}\leq 2,\ j=1,2,\cdots,26,t=2023,2024,\cdots 2029,\forall i \quad (11)$$

For irrigated land, it is not allowed to plant rice continuously for two seasons, that is:

$$y_{16jt}+y_{16j(t+1)}\leq 2,\ j=27,28,\cdots,34,t=2023,2024,\cdots 2029,\forall i \quad (12)$$

For smart greenhouses, the crops planted in the first season and the second season, as well as the second season and the following year's first season, must be different, that is:

$$\begin{aligned} &y_{ijt} + y_{i(j+28)t} \le 2,\ j = 51,52,\cdots,54, t = 2024,2025,\cdots 2030, \forall i \\ &y_{i(j+28)t} + y_{ij(t+1)} \le 2,\ j = 51,52,\cdots,54, t = 2023,2024,\cdots 2029, \forall i \end{aligned} \tag{13}$$

When irrigated land is used for growing vegetables in two seasons, only one type of vegetable among Chinese cabbage, white radish, and red radish can be planted in the second season, that is:

$$\sum_{i=35}^{37} y_{ijt} = 0 or 2,\ j \in [56,63], \forall t \tag{14}$$

The single-season cultivation of rice and the double-season cultivation of vegetables on irrigated land are mutually exclusive, that is:

$$y_{i(j+28)t} + y_{16j(t+1)} \le 2,\ j \in [27,34], t \in [2023,2029], \forall i \tag{15}$$

For non-smart greenhouses, it is required that for each plot, the sum of the planting area of each type of legume crop over three consecutive years is greater than the total area of the plot, that is:

$$\sum_{T=t}^{t+2} \sum_{i \in [1,5] \cup [17,19]} y_{ijT} \ge 2, t = 2023,2024,\cdots 2028,\ j \in [1,50] \tag{16}$$

For smart greenhouses, it is required that for each plot, over three consecutive years, the sum of the planting area of each type of legume crop for each of the two seasons per year is greater than the total area of the plot, that is:

$$\sum_{T=t}^{t+2} \sum_{i \in [1,5] \cup [17,19]} y_{ijT} + y_{i(j+28)T} \ge 2, t = 2023,2024,\cdots 2028,\ j \in [51,54] \tag{17}$$

### 3.3. Model Establishment

After the aforementioned research and analysis, the objective function and constraints derived are combined to establish a mixed-integer optimization model, which is solved using the branch and bound method [9]. The specific conclusions are presented in the following text.

## 4. Results

### 4.1. Solution Results of the First Scenario Model

After the aforementioned experimental derivation, the solution yields a maximum profit of 38,502,320 yuan for rural crop production in the case of unsold waste from 2024 to 2030. The data on the area of land used for each crop from 2024 to 2030 is shown in Figure 3:

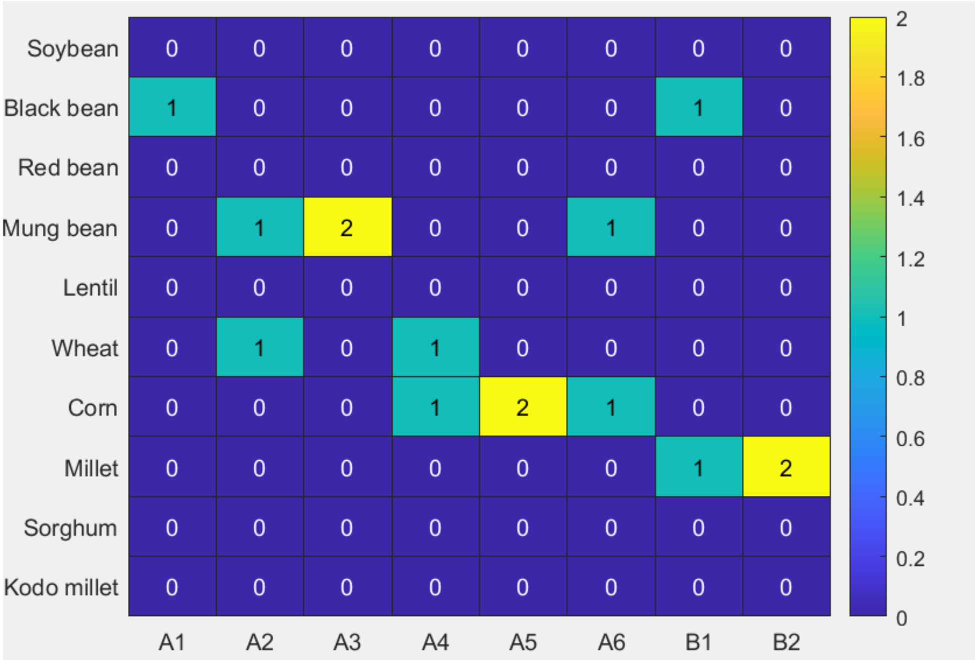


**Figure 3.** Partial Heatmap of Crop Cultivation Areas for Scenario One in 2024

In the figure, 0 indicates no cultivation, 1 indicates cultivation of half the area, and 2 indicates full cultivation (due to space limitations, only partial data for crop cultivation areas in 2024 is displayed here). From this, the optimal planting strategy for the rural area under the condition of unsold waste is derived.

### 4.2. Solution Results of the Second Scenario Model

Following the experimental derivations mentioned earlier, the solution obtained for the scenario of overproduction leading to price reduction shows that the maximum profit for rural crop production from 2024 to 2030 is 57,368,600 yuan. The data on the area of land used for each crop from 2024 to 2030 is illustrated in Figure 4:

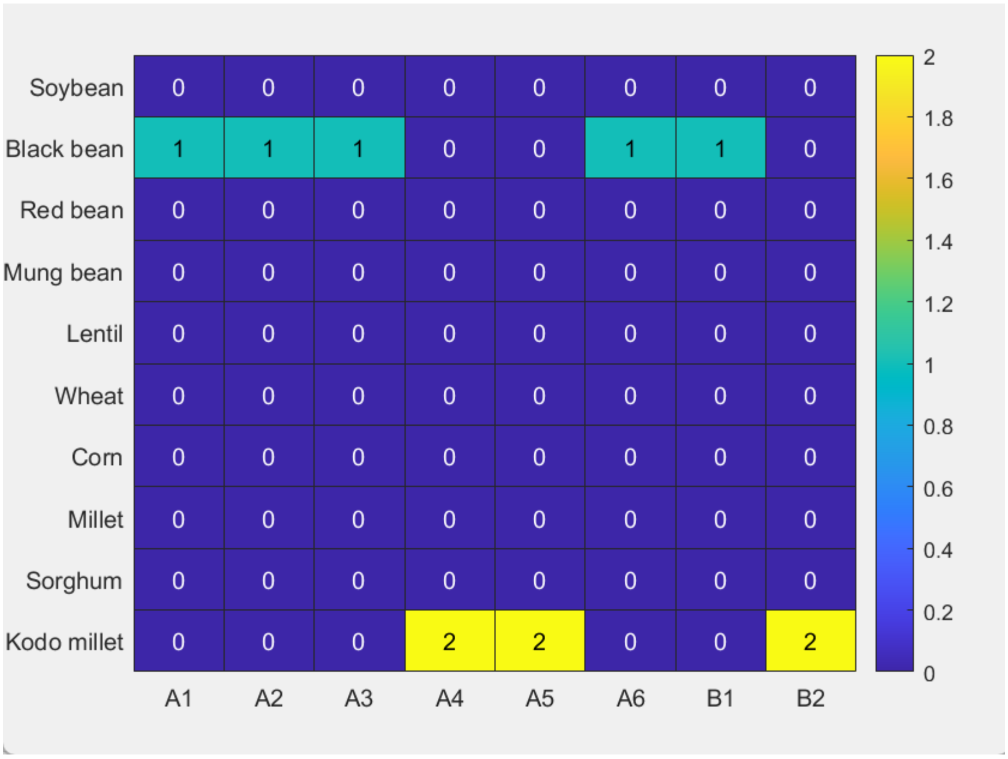


**Figure 4.** Partial Heatmap of Crop Cultivation Areas for Scenario Two in 2024

The meanings of the symbols in Figure 4 are the same as those in Figure 3. From this figure, the optimal planting strategy for the rural area under the condition of overproduction and price reduction is derived, we can plant the corresponding vegetables at the corresponding times and on the corresponding land based on the data in the picture.

## 5. Conclusions

This paper applies the branch and bound algorithm to optimize the crop planting strategies in rural areas of Northern China, addressing the issue of maximizing agricultural output on limited arable land. We developed a mixed-integer optimization model that takes into account various constraints such as crop types, plot areas, irrigation requirements, and crop rotation, with the goal of maximizing profits under two scenarios from 2024 to 2030: (1) discarding unsold crops and (2) selling surplus crops at a discount. The results indicate that under Scenario One, the maximum profit achieved is 38,502,320 yuan, and under Scenario Two, the maximum profit achieved is 57,368,600 yuan. These findings validate the effectiveness and reliability of our model in providing optimal planting strategies that align with the local agricultural context. Future work could expand the applicability of the model to other regions and refine the constraints.

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