The problem of crop planting strategies based on the stochastic linear programming model

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Abstract. This research aims to optimize crop planting strategies in specific villages within the mountainous areas of North China under the condition of limited cultivated land resources. The objective is to promote the sustainable development of rural economies while providing scientific guidance for planting decisions made by farmers and agricultural enterprises. Considering the significant impact of market fluctuations on agricultural production and the high risks faced by farmers, this study develops a stochastic linear programming (SLP) model based on 2023 data. The model analyzes optimal planting strategies over the next seven years, focusing on the annual sales growth range (5%-10%) of wheat and corn, as well as the sales, yield, planting costs, and price fluctuations of other crops. Incorporating constraints such as plot types and area limitations, the model demonstrates improved planting strategies under market fluctuations, reducing risks in extreme situations and enhancing income stability. The findings underscore the applicability of SLP models in addressing agricultural uncertainties and provide a valuable reference for achieving sustainable rural development in mountainous areas.

Keywords: Planting strategies, stochastic linear programming model, and Sustainable development.

1. Introduction

With global climate change and fluctuating market demands, optimizing agricultural planting strategies under uncertain conditions has become a significant research topic in agriculture. This rural area in the mountainous region of northern China is located in a relatively harsh environment. Despite limited arable land resources, the area boasts diverse types of farmland, including flat drylands, terraces, hillside fields, and irrigated land. These plots are suitable for growing various crops such as grains, rice, and vegetables. Additionally, the village has 16 conventional greenhouses and 4 smart greenhouses for cultivating vegetables and edible fungi. However, due to the varying suitability of crops for different plots, seasonal limitations, market demand uncertainties [1], and planting costs, scientifically and rationally allocating crop planting areas and optimizing planting strategies are crucial for promoting sustainable rural economic development.

In the field of agricultural planting strategy optimization, numerous scholars have made significant contributions. Wu et al. (2024) explored the characteristics of linear programming models in agricultural planting[2], emphasizing their advantages in resource allocation optimization, consideration of seasonality and climate factors, and multi-objective decision-making, providing valuable references for agricultural planting decisions. Zhu et al. (2006) developed a linear programming model to optimize agricultural planting structures, using constraints such as land resources and crop water requirements to propose practical and reasonable optimization plans [3]. Liu (2020) applied a multivariate linear model to study farmland water resource planning, providing strong support for improving agricultural production efficiency [4].

To address uncertainties, Xu et al. (2024) combined gray prediction and linear programming to optimize land use in the Pearl River Delta, considering the dynamic characteristics of land use changes and providing scientific grounds for rational land resource allocation [5]. Wen et al. (2024) proposed an optimization strategy based on multi-model coupling to maximize land use diversity and ecological service value, offering new insights for the sustainable development of agricultural ecosystems [6].

Additionally, Ma et al. (2021) applied a mixed-integer linear programming model to optimize fruit supply chains, improving operational efficiency and economic benefits [7]. Wang et al. (2019) studied stochastic ranking problems based on linear programming priority strategies, providing effective methods for solving complex decision problems [8]. Dong et al. (2024) proposed a continuous linear programming solution for multi-refinery production planning optimization, enhancing enterprise production management levels [9].

Nevertheless, agricultural production faces numerous uncertainties, such as climate fluctuations and market demand changes, posing significant challenges to optimizing planting strategies. Although existing studies have partially considered these factors, research on stochastic optimization under complex plot types, multi-crop rotations, and seasonal constraints remains relatively scarce. This study aims to address this gap by proposing a comprehensive planting strategy optimization model based on stochastic linear programming, incorporating diverse plot types and market fluctuation factors to maximize agricultural revenues and minimize risks in mountainous areas.

This study includes a preparatory section on relevant theories and an experimental research section, ultimately concluding that stochastic linear programming models can provide optimal planting strategies to maximize profits under various complex scenarios and market fluctuations. This serves as a critical reference for farmers and agricultural enterprises. The theoretical section focuses on the stochastic linear programming model and its constraints, explaining how it combines linear programming with uncertainty factors. The experimental research section includes an analysis of planting strategies using stochastic linear programming, applying the theoretical knowledge to detailed system modeling, and solving the model with optimization software to obtain results.

2. Relevant Theories

In this study, to address the optimization of crop planting strategies in mountainous rural areas under a fluctuating market environment, Stochastic Linear Programming (SLP) was adopted as the core model. The following provides background knowledge and the mathematical foundation related to the theories of this research, including the basic principles of the Linear Programming (LP) model and the extensions of the Stochastic Linear Programming model.

Linear Programming (LP) is a method for optimizing a linear objective function subject to a set of linear constraints [10]. In agricultural planting optimization, the objective is typically to maximize the total revenue from planting. This involves achieving the highest profit through rational crop selection and area allocation within the constraints of available land resources and planting costs.

Based on this study, a simple objective function is assumed as follows:

$$Maximize\ Z = \sum_{i=1}^{n} (p_i \times y_i - c_i) \times A_i \tag{1}$$

Where Z is the total profit, p_i is the market selling price of crop i, y_i is the yield per unit area of crop i, A_i is the planting area of crop i, and c_i is the planting cost of crop i.

In this research model, uncertainties are introduced through stochastic simulation methods. Specifically, sales volume and market price are treated as random variables following a certain distribution. For example, for a given crop i, if its sales growth rate fluctuates randomly within the interval [a, b], the annual change in sales volume can be expressed using the following formula:

$$s_{i,t+1} = s_{i,t} \times (1 + r_i) \tag{2}$$

Where $s_{i,t+1}$ represents the sales volume in year t+1, $s_{i,t}$ represents the sales volume in year t, and r_i is the stochastic growth rate of sales volume. This is used to simulate fluctuations in price and yield, typically defined as a random variable within a specific range.

Stochastic Linear Programming (SLP) is an optimization method designed for uncertain environments, particularly for scenarios involving random fluctuations in parameters. In this study, the SLP model is used to evaluate the impact of market demand and crop price fluctuations on planting decisions. Its objective function incorporates uncertainty factors to enhance the robustness of the model under fluctuating market conditions. The model's objective function is derived by integrating the above formulas (1) and (2), resulting in the stochastic linear programming objective function:

$$Maximize\ Z = \sum_{i=1}^{n} (p_i \times (1 + r_i) \times y_i - c_i) \times A_i \tag{3}$$

In practical applications, to ensure the reasonableness and feasibility of the solution, a series of constraints need to be established. These constraints include: plot type restrictions (e.g., different types of plots are suitable for different crops), crop rotation requirements (e.g., rotation of leguminous crops), seasonal restrictions (e.g., planting different crops in different seasons), and limitations on the number of plots and crops. The general form of these constraints is as follows:

$$\sum_{j=1}^{m} a_{ij} \times x_{ij} \leqslant b_i, \quad x_{ij} \geqslant 0 \tag{4}$$

Where a_{ij} represents the resource constraint coefficient, reflecting the resource requirements of crop j on plot i; b_i is the resource upper limit of plot i; and x_{ij} is the decision variable (which can be one-dimensional or higher-dimensional), indicating whether crop j is planted on plot i.

3. Experiment

This experiment aims to address the impact of uncertainties in agricultural production on planting decisions. By integrating the optimization results under stable conditions from the 2023 crop planting data with real-world uncertainty handling solutions, the study seeks to develop more robust crop planting strategies [9]. The experimental process centers on the Stochastic Linear Programming (SLP) model, incorporating stochastic simulations while fully considering real-world constraints. The goal is to derive optimal planting strategies and solutions for maximizing profits under various uncertain factors.

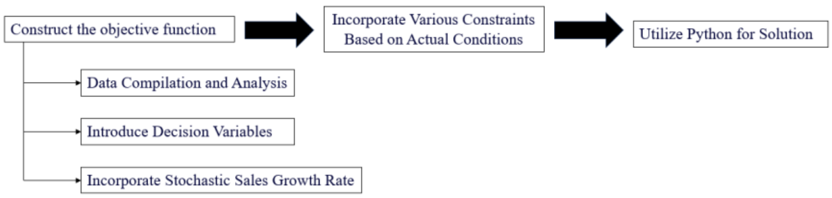

Figure 1. Implementation Process Design.

Based on Figure 1, the main objective function of the system is constructed as follows:

First, analyze the basic relationship between revenue and cost:

$$Profit = Revenue - Cost \tag{5}$$

In agricultural planting: revenue comes from crop sales, calculated as the yield per acre multiplied by the market price of the crop. Costs include planting costs per acre, such as seeds, fertilizers, labor, etc.

Second, analyze revenue and expenses. To construct the objective function, it is first necessary to clarify the components, including revenue, cost, area, and other factors. The analysis of revenue and cost is as follows. In this study, it is assumed that all yields are sold, and sales revenue comes from the yield and selling price per unit area for each crop [3]. Specifically expressed as:

$$SalesRevenue = Yield \times Price \tag{6}$$

Planting costs are the costs per acre, including fertilizers, labor, water resources, etc. Specifically expressed as:

$$PlantingCost = CostperAcre \times TotalAcres \tag{7}$$

The total planted area affects both the total revenue and total cost. The impact of land parcel area is reflected in both sales revenue and planting cost. Thus:

$$TotalRevenue = Yield \times Price \times LandParcelArea \tag{8}$$

$$TotalCost = CostperAcre \times LandParcelArea \tag{9}$$

Finally, introduce decision variables. In agricultural planting, different land parcels can choose to grow different crops in different seasons. Therefore, a three-dimensional decision variable is introduced $x_{i,j,t}$ .It represents whether crop j is planted on land parcel i during season t.

- If planted, then:

$$x_{i,j,t} = 1 \tag{10}$$

- If not planted, then:

$$x_{i,j,t} = 0 \tag{11}$$

The objective function needs to incorporate this decision variable to control whether a specific crop is planted on a particular land parcel in a given season. This ensures that only the selected crop in each parcel and season generates revenue and incurs costs [4-7].

The stochastic simulation method is a random sampling approach that uses random numbers or probabilistic statistics to solve computational problems. It is particularly suitable for problems that cannot be solved using exact mathematical methods. Its core idea is to estimate solutions through a large number of random trials. This method is often applied to approximate solutions for complex systems, integration, optimization, and probability problems.

In this study, the uncertainties include the following aspects:

1) The annual growth rate of future sales for wheat and corn is expected to range from 5% to 10%.

2) The annual sales volume of other crops is expected to fluctuate by approximately ±5% compared to 2023.

3) The yield per acre of crops generally fluctuates by around ±10% annually.

Stochastic simulation was used to simulate the sales volume, yield, planting cost, and selling price of various crops. This allows the model to generate different planting plans under various uncertainty conditions, compare profitability, and determine the optimal planting strategy [8].

To account for data fluctuations, the concept of stochastic sales growth rate was introduced. The primary purpose is to incorporate uncertainty and variability, simulating scenarios where the sales volume or price of crops may increase or decrease each year. This kind of uncertainty is common in real-world problems, especially in agricultural planting, where crop demand and market conditions can vary annually. By assigning random sales growth rates to different crops, this approach more realistically reflects such uncertainties and integrates them into the optimization model.

Based on formula (2), the stochastic sales growth formula can be derived:

$$s_{i,t+1} = s_{i,t} \times (1 + r_{i,t}),\ r_{i,t} \epsilon [a, b] \tag{12}$$

Where $s_{i,t+1}$ represents the sales volume for year t+1, $s_{i,t}$ represents the sales volume for year t, and $r_{i,t}$ is the stochastic growth rate for year t. For example, the sales volume of wheat and corn fluctuates within the specified range [0.05, 0.10].

After introducing the stochastic sales growth formula, its fluctuation can be incorporated into the objective function based on formula (1). The specific form is shown in formula (3). On this basis, it is assumed that all annual production can be sold within the same year. Therefore, the objective function for this study can be derived from formula (3) and formula (12):

$$Maximize\ Z=\sum_{i=1}^{n}(p_j\times y_j\times(1+r_{i,t})-c_i)\times A_i\times x_{i,j,t} \tag{13}$$

Based on the above objective function, important constraints in the agricultural planting process are incorporated as follows:

To avoid planting the same crop on too many plots, a constraint is set to limit the planting quantity of each crop in each season to not exceed a certain value [5].

The following constraints are established based on practical considerations and formula (4): flat drylands, terraced fields, and slopes can only grow one season of grain crops annually. Irrigated land can either grow one season of rice or two seasons of vegetables, but not both at the same time. Greenhouses can grow one season of vegetables and one season of edible fungi (ordinary greenhouses) or two seasons of vegetables (smart greenhouses). Each plot must grow legumes at least once every three years to prevent excessive soil nutrient depletion and maintain soil fertility. Certain plot types, such as irrigated land or greenhouses, have specific crop planting restrictions. For example, greenhouses are suitable for vegetables, while irrigated land is suitable for rice or vegetables. These constraints ensure the compatibility between crop types and plot types.

For grain crop constraints:

$$\sum_{j\in Grain\ category}x_{i,j,t}\leqslant 1 \tag{14}$$

For the constraints on irrigated land:

$$\sum_{j\in Rice}x_{i,j,t}+\sum_{j\in Vegetable\ category}x_{i,j,t}\leqslant 1 \tag{15}$$

As for vegetable type j:

$$x_{i,j,1}=x_{i,j,2} \tag{16}$$

Ordinary greenhouse:

$$\sum_{j\in Edible\ fungi\ category}x_{i,j,t}+\sum_{j\in Vegetable\ category}x_{i,j,t}\leqslant 1 \tag{17}$$

Smart greenhouse:

$$\sum_{j\epsilon Vegetable\ category}x_{i,j,1}=1 \tag{18}$$

Crop rotation requirements for legumes:

$$\sum_{t\epsilon 1,3,5}\sum_{j\epsilon Legume\ crops}x_{i,j,1}=1 \tag{19}$$

Crop planting restrictions for specific plots:

$$\sum_{j\epsilon Vegetable\ category,Legume\ crops}x_{i,j,1}=1 \tag{20}$$

After determining the objective function, decision variables, and constraints, the model was solved using Python software.

Through the detailed systematic modeling process described above, Python was used to obtain the optimal planting plan under the condition of maximizing profitability. This included accounting for various complex scenarios and sales fluctuations, as detailed in the following sections.

4. Results

Through the construction and experimental analysis of the Stochastic Linear Programming (SLP) model, combined with crop planting data from a village in the mountainous areas of North China in 2023 (sourced from local agricultural statistics and farmer surveys), multi-scenario simulations were conducted to address the uncertainties of market demand and price fluctuations. The following research findings were obtained:

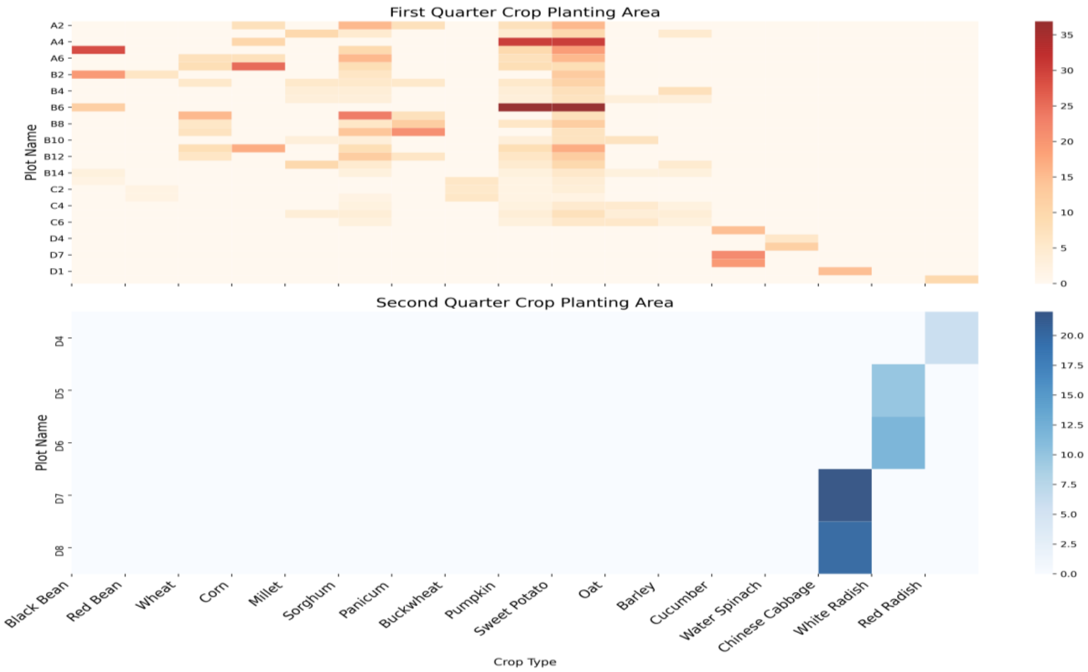

Figure 2. Optimal Agricultural Planting Strategy for Rural Areas in 2025.

Figure 2 illustrates the agricultural planting structure for the first and second quarters of 2025 under market fluctuation conditions, as determined by the Stochastic Linear Programming (SLP) model. The results indicate that over a seven-year period, by adhering to various constraints such as crop rotation, seasonal, and land constraints, and incorporating fluctuation scenarios, it is possible to achieve an optimal agricultural planting strategy that maximizes revenue while effectively utilizing resources.

Table 1. Revenue from Optimal Planting Strategy.

Type	Grain	Vegetable	Edible Fungus
Profit from production in Q1 2024	1590018.213	1474098.661	128857.0585
Profit from production in Q2 2024	860.228254	718998.7247	239130.5388
Profit from production in Q1 2025	1393766.107	1660756.66	0
Profit from production in Q2 2025	0	648484.2487	319928.5568
Profit from production in Q1 2026	1463492.239	1339571.328	106790.3367
Profit from production in Q2 2026	940.4874206	717611.6666	304961.1902
Profit from production in Q1 2027	1702363.678	1245080.414	0
Profit from production in Q2 2027	4735.341944	819565.7615	217440.1718
Profit from production in Q1 2028	1390925.875	1069163.185	0
Profit from production in Q2 2028	1699.715873	427486.9273	243348.967
Profit from production in Q1 2029	1513103.85	1430763.559	0
Profit from production in Q2 2029	1309.26	218018.5632	243698.5503
Profit from production in Q1 2030	1727692.189	738637.9163	4931.975
Profit from production in Q2 2030	1048.99623	1213283.356	297484.8068

Table 1 presents the revenue generated from the optimal planting strategy, highlighting the total revenue under uncertain risks and fluctuations. A comparison with the 2023 data reveals that even under various constraints and uncertainties, relatively high revenue can still be achieved. Compared to scenarios without considering fluctuations, the Stochastic Linear Programming (SLP) model effectively reduces revenue fluctuation risks, with a reduction in revenue variance of approximately 20%. This validates the model's adaptability to different price and sales fluctuation conditions.

From the above research and analysis, it can be concluded that the SLP model effectively optimizes crop planting strategies in mountainous rural areas. It not only enhances revenue stability but also mitigates market fluctuation risks, providing scientific guidance and theoretical support for the sustainable development of mountainous agriculture.

5. Conclusion

In this study, the optimization challenges of crop planting strategies in a specific village in the mountainous areas of North China under limited arable land resources were systematically analyzed to address the complex uncertainties of market demand and price fluctuations. Through the rigorous construction of a Stochastic Linear Programming (SLP) model, real-world factors such as plot characteristics, crop rotation requirements, seasonal constraints, and market dynamic variables were skillfully integrated. Using 2023 planting data, the model successfully derived multi-scenario optimal planting plans, achieving a balance between maximizing revenue and minimizing risks. Experimental results clearly demonstrate the model's excellent performance in complex and diverse scenarios, significantly enhancing revenue stability, substantially reducing market fluctuation risks, and showcasing robust advantages over traditional strategies. This study firmly establishes the reliability and broad adaptability of the SLP model in the field of agricultural planting strategy optimization, providing a solid scientific foundation for sustainable agricultural development in mountainous regions.

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