Research On Optimization and Uncertainty Analysis of Crop Cultivation Based on The Monte Carlo Algorithm

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Abstract. With population growth and climate change, agriculture faces challenges such as limited land, fluctuating market demand, and rising costs. This study focuses on optimizing crop yields and profits through scientific forecasting in uncertain environments. Using 2023 data on various crops and plot types, an integer programming model is developed to maximize profit, considering crop growth patterns, environmental constraints, and a requirement to plant legumes every three years to maintain soil fertility. The model incorporates non-negative and binary variables, along with quarterly constraints, to perform 0-1 programming. A Particle Swarm Optimization algorithm is used for iterative calculations, while the Monte Carlo method generates random inputs to account for uncertainty and reduce error. By analyzing different scenarios, the model identifies optimal planting strategies and predicts cropping plans for the next six years, providing a robust framework for decision-making in agricultural production.

Keywords: Binary Processing, Particle Swarm Algorithm, Monte Carlo Algorithm, 01 Planning.

1. Introduction:

As the global population agricultural production is under unprecedented pressure. With limited land resources and changing climatic conditions, increasing crop yields, optimizing planting patterns, and reducing planting costs have become important issues in agricultural production. At the same time, the volatility of market demand, the unpredictability of natural disasters, and the rising cost of production have all contributed to a significant increase in the uncertainty of agricultural output. To cope with these challenges, how to maximize profits with limited resources and reduce the impact of uncertainty on agricultural production through scientific forecasting methods has become a key issue in agricultural management and decision optimization. In recent years, with the development of data science and technology, modeling and prediction of uncertainty factors using algorithms such as Monte Carlo simulation have opened up new possibilities for decision-making in agricultural production.

For the study of crop cultivation optimization, we proceeded to use the Monte Carlo algorithm. Introducing the proposed Monte Carlo algorithm and the simulation of two random variables with correlation used in "Simulation of Correlated Random Variables in Uncertainty Evaluation Based on Monte Carlo Methods" and "Some Studies on Monte Carlo and Monte Carlo-like Methods", which are mainly used in numerical methods and are widely used in various fields such as pharmaceuticals, aerospace, and geology, and carbon ion therapy, The Monte Carlo algorithm is used for optimization in various fields such as carbon ion therapy, assessment of the potential of "Tibetan power transmission" and quantitative uncertainty analysis of the seismic response of high concrete dams.

In this study, we set up an objective function optimization model with the highest total profit as the objective, and binarised the decision variables with different plots, crops, seasons, and years, if the binarised decision variables and the number of planting of each specific crop are considered jointly, this model will become a non-linear non-convex optimization model and cannot be solved. So this study set the decision variable as a purely binary decision variable, for the desired result, the binary decision variable of the optimization result can be processed again to normalize all crops to the intrinsic value. The planting patterns of different plot types, crop growth patterns, and planting areas in the topic are used as constraints, and then the objective function is optimized to find the optimal solution. The prediction problem with uncertainty conditions is added on the basis of the objective function and constraints, and uncertainty modeling is carried out by considering the uncertainties in the expected crop sales volume, acreage yield, planting cost, and sales price as well as potential risks. Repeated random sampling of data at different growth rates using the Monte Carlo algorithm, assuming that the Monte Carlo sampled data conforms to a normal distribution, optimization of the objective function, calculation of returns under different scenarios, and ultimately, calculation of the maximum return under fluctuating scenarios.

2. Data Sources and Visualisation:

Data for this article was obtained from https://www.mcm.edu.cn/.

2.1. Data processing

Assuming that the expected sales volume, cost of cultivation, acreage, and sales price of various crops remain stable relative to 2023, different worksheets of Annex 1 and Annex 2 are fused, using the cultivation and sales data of 2023 as the base input data for the model, and pre-processing the acreage, cost of cultivation, and expected sales volume of different crops. The average of the sales price interval was taken as the sales unit price, the random volume screening was done using the Monte Carlo algorithm, and its average was calculated and found to be close to the average of the sales unit price interval, so the average of the sales price interval was taken as the sales unit price.

2.2. Data visualisation

The relationship between different planting areas and acreage yield is shown in Figure 1.

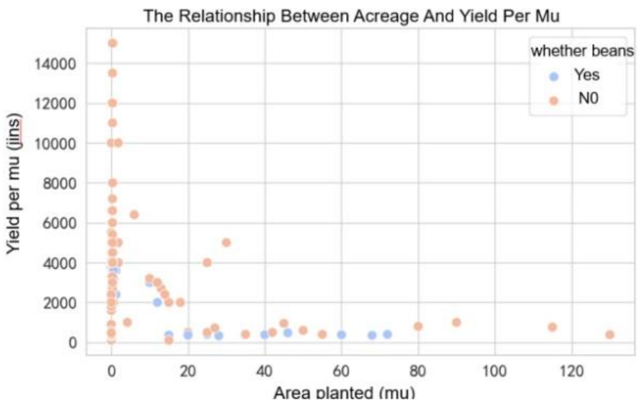

Figure 1 Planted area and yield per acre

The distribution of unit sales prices for different crop types is shown in Figure 2, with the highest unit sales prices for edible mushrooms, lower unit sales prices for food, and the lowest unit sales prices for vegetables.

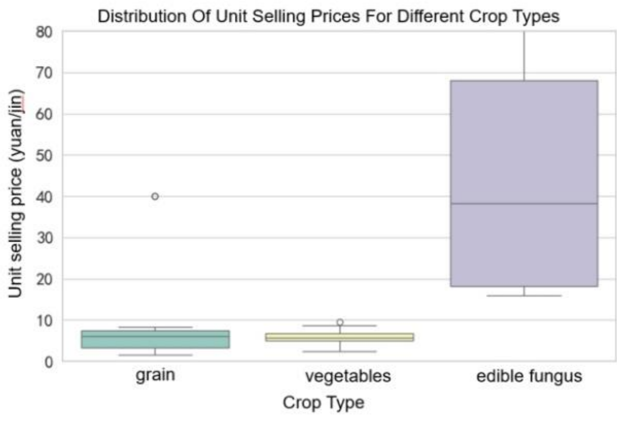

Figure 2 Unit price of sales

The area of crops grown in different seasons is shown in Figure 3, with the largest area of food grown in a single season, vegetables grown in both the first and second seasons, edible mushrooms grown only in the second season, and the smallest area of cultivation.

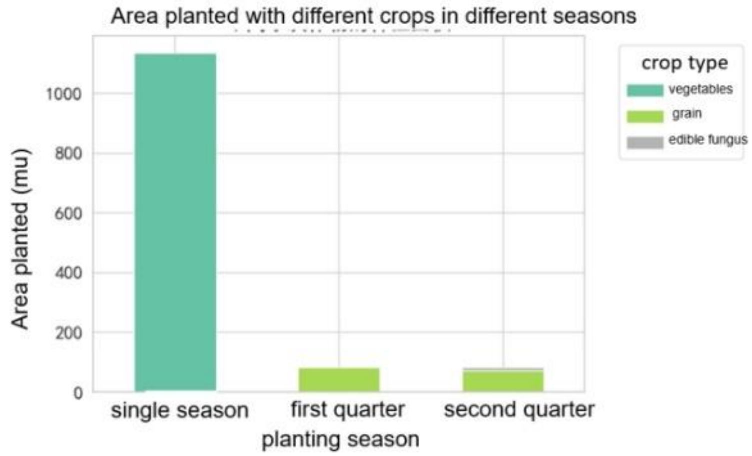

Figure 3 Planted area

The total and average profits for different crop types are shown in Figure 4, which shows that edible mushrooms have the lowest total profit but the highest average profit, food grains have the highest total profit, and vegetables have the lowest average profit.

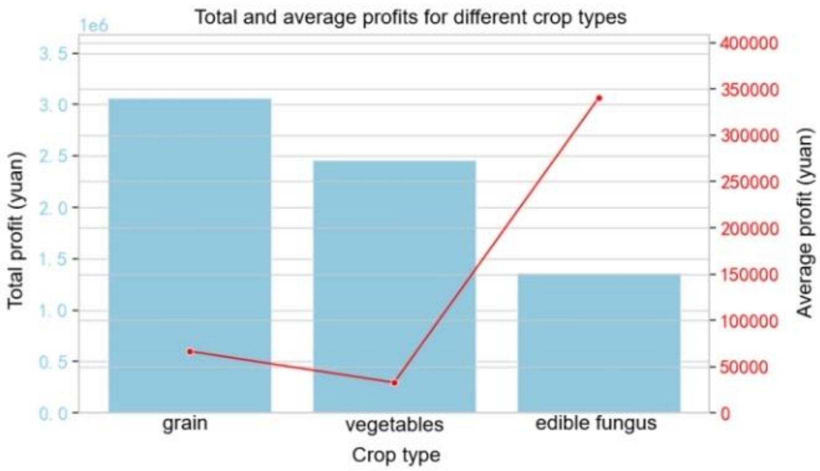

Figure 4 Total Profit and Average Profit

3. Profit Maximization and Integer Programming in Crop Planting:

The mathematical model based on integer programming establishes profit maximization as the objective function and, in the solution process, is binarised to normalize to a fixed value, simplifying the calculations and optimizing its functional model.

This study assumes that the sales volume of each crop will fluctuate, that is, the market demand will be more or less, upholding the principle that it is better to waste, but also must satisfy the people's food needs, the emergence of the total production of crops exceeds the expected sales volume, resulting in stagnant sales of and reduced sales of two kinds of treatment, that is, the objective function of the two problems is calculated by different formulas, the constraints are the same.

For the next n years, only the last three years need to be optimized to ensure that pulses will be planted every three years, i.e. to ensure that the land is nutritious, and finally measured by the objective function of profit maximization, and for the next n years, the results of the optimization need to be filled in by repeating the process only once. In order to make the study more realistic, two scenarios are assumed here in this study: Scenario 1 is a situation where the total crop production exceeds the expected sales volume, resulting in a stagnant sale; and Scenario 2 is a sale at a reduced price

3.1. Decision-making variables

Define the decision variable as, $x_{i,j,t}$, i denotes as the i th plot, $i \in \{1,2,...N\}$; N=34, j denotes as th j the crop, $j \in \{1,2,..., M\}$; t denotes as the t th season, $t \in \{1,2,...6\}$.

3.2. Objective function

The objective is to maximize the total return on the crop, the

For case 1, the maximum return for each crop in the first t season can be expressed as, the

$$Max\sum_{j=1}^{M}\sum_{t=1}^{6}(\min(\sum_{i=1}^{N}x_{i,j,t}\cdot Y_{ij}, S_{P_{ij}})\cdot P_j - C_{ij}\cdot x_{i,j,t}) \tag{1}$$

For case II, the maximum return for each crop j in quarter t can be expressed as.

$$Max\sum_{j=1}^{M}\sum_{t=1}^{6}(\min(S_{P_{IJ}}, \sum_{i=1}^{N}x_{i,j,t}\cdot Y_{ij}) + \max(0, \sum_{i=1}^{N}x_{i,j,t}\cdot Y_{ij} - S_{p_{ij}}))\cdot P_j - C_{i,j}\cdot x_{i,j,t} \tag{2}$$

$\sum_{i=1}^{N}x_{i,j,t}\cdot Y_{ij}$ denotes the yield for all lands and $C_{i,j}\cdot x_{i,j,t}$ denotes the cost of cultivation.

3.3. Constraints

non-negative and binary variable constraints, the decision variable $x_{i,j,t}$ is binary, and only one crop is grown on the same plot in the same season.

1, Plot i grows crop j in season t

0, Plot i did not grow crop j in season t (3)

Flat drylands, terraces, and hillsides can only grow food crops in one season and cannot be grown in a second season, assuming that the set of food crops is S_{grain}, $j \in S_{grain}$ and that the jth crop is planted in season t in the ith plot as 1, and not planted as 0, then.

$$\sum_{j\in S_{grain}} x_{i,j,1} = 1 \tag{4}$$

$$\sum_{j=1}^{M} x_{i,j,2} = 0 \tag{5}$$

Only one season of rice or two seasons of vegetables can be grown in watered land and only specific vegetables can be grown in the second season, assuming that the set of crops in watered land is S_{rice}, $j \in S_{rice}$ Assuming that the set of vegetable crops is S_{vega}, $j \in S_{vega}$, if food is grown in the 1st season then vegetables are grown in the 2nd season or vegetables are grown in the 1st season and vegetables continue to be grown in the 2nd season, the

$$\sum_{j\in S_{rice}} x_{i,j,1} + \sum_{j\in S_{vega}} x_{i,j,1} = 1 \tag{6}$$

$$\sum_{j\in S_{rice}} x_{i,j,1} + \sum_{j\in S_{vega}} x_{i,j,2} = 1 \tag{7}$$

Ordinary greenhouses grow one season of vegetables and one season of edible mushrooms per year, let the set of edible mushroom crops be S_{jun}, $j \in S_{jun}$, plant vegetables in the first season and edible mushrooms in the second season or plant edible mushrooms in the first season and edible mushrooms in the second season, then.

$$\sum_{j \in S_{rice}} x_{i,j,1} + \sum_{j \in S_{jun}} x_{i,j,1} = 1 \tag{8}$$

$$\sum_{j \in S_{vega}} x_{i,j,2} + \sum_{j \in S_{jun}} x_{i,j,2} = 1 \tag{9}$$

Smart greenhouses must grow two seasons of vegetables.

$$\sum_{j \in S_{vega}} x_{i,j,1} = 1 \tag{10}$$

$$\sum_{j \in S_{vega}} x_{i,j,2} = 1 \tag{11}$$

Pulses are rotated once every three years, assuming that the set of pulses is S_{bean}, $j \in S_{bean}$ and those pulses must be planted in one of the six seasons.

$$\sum_{j \in S_{bean}} \sum_{t=1}^{6} x_{i,j,t} > 0 \tag{12}$$

Each crop should not be grown in successive heavy crops on the same plot (including greenhouses).

$$x_{i,j,t} \cdot x_{i,j,t-1} = 0; t = 2,4,6 \tag{13}$$

Crops should not be spread out, with plots i a i' djacent to each other, limiting the area of the same crop to adjacent plots, and η being the allowable acreage difference.

$$|x_{i,j,k} - x_{i',j,k-1}| \leq \eta \tag{14}$$

Finally, the global optimization was performed by particle swarm algorithm and solved using Python and the results are shown in Table 1.

Table 1 Planting programme 1(partial)

	soya bean	black bean	white mushroom (Pleurotus eryngii)	morel mushrooms
first quarter	1	0	0	1
	......	......	......	......
	0.62	0.54	1	0
first quarter	0	1	1	1
	......	......	......	......
	1	0.78	1	1

4. Stochastic Modelling and Yield Prediction for Crop Planting:

As a forecasting problem, with expected crop sales, acreage, planting costs, and sales price fluctuations, predict the planting for the next six years, based on the previous model, using the Monte Carlo algorithm for random sampling to cut down the uncertainty, and ultimately calculate the average return.

According to the actual situation, the sales volume of crops is affected by climate, market demand, transport situation, trade policy, and other circumstances, generally speaking, in non-extreme circumstances, the sales volume of crops conforms to the normal distribution; acreage yield of crops is mainly affected by climate, soil, pests and diseases and other natural factors, which conforms to the normal distribution, and in the case of natural disasters, there may be a large deviation; the crop Cultivation costs are affected by market prices, labor costs, in the long run, in addition to inflation, in line with the normal distribution; crop sales price fluctuations are affected by market demand, costs, policies and other factors, fluctuations are large, mainly by crop type classification, vegetables, food, edible fungi, respectively, food prices tend to be stable, the price of vegetables has risen, the price of edible fungi fell, in line with the normal distribution. Normal distribution.

4.1. Decision-making variables

$x_{i,j,k}$, i denotes the first i plot, $i \in$ {1,2,...N};N=34, j denotes the first j crop, $j \in$ {1,2,...,M}, k denotes the first k year, k ={2023,...,2030}.

4.2. Introducing random variables and modeling uncertainty

Changes in crop sales, with sales of wheat and maize growing at 5-10 percent per year.

$$D_j(k) = D_j(2023)(1+r_j^k)^{(k-2023)} (r_j^k \in [0.05, 0.1]) \tag{15}$$

Sales of other crops vary by ± 5 percent per year.

$$D_j(k) = D_j(2023)(1+r_j^k)^{(k-2023)} (r_j^k \in [-0.05, 0.05]) \tag{16}$$

Acreage changes by ± 10 percent per year, and the

$$\mathrm{A}_j(k) = A_j(2023)(1+u_j^k)^{(k-2023)} (u_j^k \in [-0.1, 0.1]) \tag{17}$$

The cost of cultivation is increasing by about 5 percent per year.

$$B_{j,k} = 1.05^{(k-2023)} C_{i,j,k} \square x_{i,j} \tag{18}$$

(a) The sale prices of crops, with food prices stabilizing and vegetable prices increasing by 5 percent per year.

$$p_j(k) = 1.05^{(k-2023)} p_j(2023) \tag{19}$$

Prices of edible mushroom crops have been declining by 1-4 percent per year.

$$p_j(k) = p(2023)(1+y_j^k)^{k-2023} (y_j^k \in [-0.01, 0.04]) \tag{20}$$

especially the morel mushrooms, which are declining by 5 percent per year.

$$p_j(k) = 0.95^{(k-2023)} p_j(2023) \tag{21}$$

Monte Carlo simulation models uncertainty in expected crop sales, acreage, planting costs, and selling prices through repeated random sampling. Again, the fluctuation intervals of each variable conform to a normal distribution, and the input data are coupled with random noise to analyse the performance of the decisions by randomly generating different scenarios.

$$\mathrm{f}\delta(r_j^k) = f(r_j^k) + \delta(2Ri - 1) f(r_j^k) \tag{22}$$

where $\mathrm{f}\delta(r_j^k)$ is the added noise data, δ is the relative error level, and Ri is a uniformly distributed random number on the interval [0,1].

4.3. Objective function

The objective function with profit maximization as the goal, E[] denotes the different doubly stochastic expectation values of the

The maximum return in year k for each crop j in case 1 can be expressed as.

$$MaxE(\sum_{k=2024}^{2030}\sum_{j=1}^{M}(\min(\sum_{i=1}^{34}x_{i,j,k}\cdot A_j(k),D_j(k))\cdot p_j(k)-x_{i,j,t}\cdot B_{j,k}) \tag{23}$$

The maximum return in year k for each crop j in case II can be expressed as.

$$MaxE[\sum_{k=2024}^{2030}\sum_{j=1}^{M}(\min(D_j(k),\sum_{i=1}^{34}x_{i,j,k}\cdot A_j(k)-D_j(k)))\cdot p_j(k)-x_{i,j,t}\cdot B_{j,k}] \tag{24}$$

4.4. Constraints

The limitation on the area of cultivated land, whereby the area cultivated on each plot is less than or equal to the actual area.

$$x_{i,j,k}\le S_i \tag{25}$$

Pulses must be planted every three years.

$$\sum_{k=2024}^{k+2}\sum_{j\in S_{bean}}x_{i,j,k}\ge \varepsilon \tag{26}$$

The same crop cannot be grown consecutively on the same plot of land.

$$x_{i,j,k}\cdot x_{i,j,k-1}=0 \tag{27}$$

Crop growing conditions, flat dry land, terraces, and slopes are suitable for growing food crops in a single season per year, watered land can be used for growing rice or vegetable crops per year, and greenhouses can be used for growing vegetables and edible mushrooms.

$$x_{i,j,k}\text{ , if j is not suitable for crops} \tag{28}$$

Crops should not be spread out, with plots i a i' djacent to each other, limiting the area of the same crop to adjacent plots, and λ being the allowable acreage difference.

$$|x_{i,j,k}-x_{i',j,k-1}|\le\lambda \tag{29}$$

Ordinary greenhouses can only grow two crops per year, vegetables and mushrooms.

$$x_{i,j,k}=0(j\notin S_{vega},j\notin S_{jun}) \tag{30}$$

Smart greenhouses can only grow vegetables every year.

$$x_{i,j,k}=0(j\notin S_{vega}) \tag{31}$$

The corresponding planting scenarios that were finally obtained by solving using Python are shown in Table 2. Among them, the mung bean planting area in 2024 was 35.6 acres, the mung bean planting area in 2026-2027 was 20 and 55 acres, respectively, and then fell back to 20 acres in 2030. Buckwheat was planted on 20 acres in 2024, 20 acres in 2027, and 14.3 acres by 2029. Corn will be planted on 80 acres in 2026 and again on 80 acres in 2030.

Table 2 Planting programme 2

vintages	strips	crop	area (of a floor, piece of land, etc)
2024	Lot 1	mung bean	35.59375
......	......	......	......
2024	Lot 26	buckwheat	20
2025	Lot 1	barley	80
......	......	......	......
2027	Lot 26	buckwheat	20
2028	Lot 1	sorghum	80
......	......	......	......
2028	Lot 26	mung bean	20
2029	Lot 1	mung bean	0.7
......	......	......	......
2029	Lot 26	buckwheat	14.315789
2030	Lot 1	sorghum	80
......	......	......	......
2030	Lot 26	mung bean	20

5. Conclusions

This study for the year 2023 for different types of plots of different crops of the data is summarised, and then based on different situations, the establishment of the maximum profit as the goal of the objective function model, and with different types of plots of different crops of the growth pattern as well as the limitations of the external environmental factors, the use of non-negative and binary variables as well as quarterly constraints on the decision-making variables, 01 planning, the establishment of the constraints, and because the three years must have one year to grow a leguminous food crop, the model only needs to be optimized for the last three years to keep the soil nutritious for continued cultivation, measured in terms of maximum profit, simplifying the calculations.

This study combines the actual situation, the expected sales volume of crops, mu yield, planting costs, and sales price fluctuation interval in line with the normal distribution, the fluctuation interval is reflected by using the expression, and the random numbers are extracted by Monte Carlo algorithm and brought into the model for operation, to cut down the uncertainty and reduce the error. Then the maximum profit under different circumstances as the goal of the mathematical model, the use of year constraints decision variables, and again set up constraints, the use of particle swarm algorithm for iterative calculations.

The application of Monte Carlo algorithms in agriculture has a wide range of prospects to help solve various uncertainty problems in agricultural production and improve the efficiency and sustainability of agricultural production. By simulating and optimizing various factors in the agricultural production process, Monte Carlo algorithms provide a powerful decision-support tool for farmers, agricultural scientists, and policy makers.

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