Optimization Model for Crop Planting Scheme Based on Mixed-Integer Programming

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Abstract. In response to the rural revitalization strategy, this paper analyzes and models relevant data based on the crop planting optimization model, aiming to formulate the optimal crop planting scheme for a village in North China. Firstly, data fitting and visualization were carried out, including creating bar charts, frequency distribution histograms, and normal fitting curves of crop yields. Next, the mixed integer programming model was used to optimize and solve the maximum profit and total profit from 2024 to 2030 in the case of overproduction and unsalable surplus. In addition, a sensitivity analysis of data perturbation was conducted for the planting cost to explore the impact of changes in the planting cost on profit, demonstrating that the model has good robustness and stability. This indicates that even when the planting cost is uncertain, the model can maintain stable profit predictions, thus providing effective decision support for crop planting.

Keywords: Mixed-Integer Programming Model, Crop Planting Optimization, Sustainable Agricultural Development, Robust Optimization Methods.

1. Introduction

Against the backdrop of the rural revitalization strategy [1], the rational utilization of arable land resources and the development of organic farming [2] are crucial for the sustainable development of rural economies [3]. A village located in the mountainous region of North China faces limited arable land resources and year-round low temperatures. With various types of open fields and greenhouses, it needs to select suitable crops and optimize planting strategies [4] to enhance production efficiency and reduce planting risks, providing a practical context for optimizing crop planting plans.

Currently, there have been significant achievements in the research on crop planting optimization. Many scholars have utilized different methods for modeling and analyzing planting plans. For instance, some studies have employed linear programming methods [5] to determine crop planting structures, aiming for efficient resource utilization. Others have constructed decision-making models based on the analysis of crop growth environments and market demands to improve planting benefits. However, these studies still have certain limitations. Some models lack sufficient accuracy and stability when dealing with complex real-world situations, such as the combined effects [6] of multiple uncertainty factors. Additionally, some research has not adequately considered the complementarity and substitutability among crops, as well as their associations with market factors [7], leading to less comprehensive optimization of planting strategies.

In light of these deficiencies in previous research, this study holds significant importance. We aim to create a more robust crop planting plan optimization model based on mixed integer programming [8] and robust optimization methods [9]. By comprehensively considering the complementarity and substitutability [10] among crops, along with the uncertainties of various factors such as yield, cost, and price, we can develop more precise planting plans. In addition, we have also conducted an in-depth sensitivity analysis [11] on the impact of planting costs on profits. This will enhance agricultural productivity, reduce risks, and provide stronger support for the sustainable development of rural economies, thereby promoting a more scientific and rational decision-making process in crop planting.

2. Mixed-Integer Programming Model

2.1. Model Purpose

This mixed-integer programming model is specifically designed to address the complex situation in crop planting where the total yield exceeds the expected sales volume, resulting in unsold surplus. Its core objective is to provide farmers with a scientific and reasonable planting scheme through precise mathematical calculations and logical analysis, aiming to maximize planting profits. In actual agricultural production, the issue of unsold goods often leads to product backlog, causing economic losses. This model effectively reduces such losses, optimizes resource allocation, and enhances the overall efficiency of agricultural production, assisting farmers in making informed decisions amid market uncertainties.

2.2. Model Establishment Process

2.2.1 Definition of Decision Variables

$x_{i,j,t}$: Clearly represents the planting area of crop$j$on specific plot i in year t. This provides foundational data for subsequent calculations of planting costs, yields, and profits.

$y_{i,j,t}$: As a binary variable, it reflects the binary nature of planting decisions. When $y_{i,j,t} = 1$, it indicates that crop j is chosen to be planted on plot i in year t; when $y_{i,j,t} = 0$, it indicates that no planting occurs, which helps to handle the discrete nature of planting choices in the model.

$z_{i,j,t}$: Characterizes the actual sales volume of crop j on plot i in year t, which is crucial for calculating sales revenue and assessing the actual benefits of the planting scheme.

2.2.2 Determination of the Objective Function

The objective function is constructed based on the principle of maximizing economic returns.

$$Maximize \sum_{i \in plots, j \in \text{crops}, \text{t} \in \text{years}}^{\Sigma} \sum (G - F), \tag{1}$$

where, $G = p_{j,2023} \times z_{i,j,t}$ represents sales revenue, $p_{j,2023}$ is the average selling price of crop j in 2023 (assuming prices remain relatively stable), and $z_{i,j,t}$ is the actual sales volume, which takes the minimum value between expected yield and expected sales volume, considering that actual sales may be constrained by market demand. $F = c_{j,2023} \times x_{i,j,t}$ represents planting costs, $c_{j,2023}$ is the planting cost of crop j in 2023, and $x_{i,j,t}$ is the planting area. By calculating the difference between sales revenue and planting costs and summing this across all plots, crops, and years, the model aims to identify the combination of planting schemes that maximizes total returns.

2.2.3 Establishment of Constraint Conditions

The total planting area does not exceed the plot area:

$$\sum_{j \in \text{crops}} x_{ijt} \le A_i, \tag{2}$$

where A_i is the plot area.

Specific planting for flat and dry land:

$$\sum_{j=1}^{15} x_{ijt} \le A_i \tag{3}$$

and

$$\sum_{j \notin \{1,2 \cdots 15\}} x_{ijt} = 0. \tag{4}$$

Irrigated land planting rules:

$$\sum_{j=16} x_{ijt} \leq A_i \tag{5}$$

and

$$\sum_{j=17}^{30} x_{ijt} \leq 2A_i. \tag{6}$$

General greenhouse planting restrictions:

$$\sum_{j=17}^{27} x_{ijt} \leq A_i \tag{7}$$

and

$$\sum_{j=38}^{41} x_{ijt} \leq A_i. \tag{8}$$

Actual sales volume constraints:

$$z_{ijt} \leq EY_{ijt} \tag{9}$$

and

$$z_{ijt} \leq ES_{ijt}, \tag{10}$$

where $EY_{ijt} = x_{ijt}Y_{jt}$ and Y_{jt} are yield per mu and ES_{ijt} is the expected sales volume.

Crop rotation constraints:

$$y_{ijt} + y_{ij,t+1} \leq 1. \tag{11}$$

Legume crop constraints:

$$\sum_{j \in \{1,2,3,4,5,17,18,19\}} y_{ijt} \geq 1. \tag{12}$$

Minimum planting area constraints:

$$M \cdot y_{ijt} \leq x_{ijt} \leq 1000 \cdot y_{ijt}, \tag{13}$$

where $M = 0.3$.

These formulas constitute a complete mixed-integer programming model designed to optimize planting schemes in the context of unsold crops. In practical applications, by inputting relevant data and utilizing mathematical optimization algorithms to solve this model, one can obtain planting schemes that maximize planting profits. This includes decision-making information such as the area allocation for various crops in different plots across different years.

The model flowchart is shown in Figure 1.

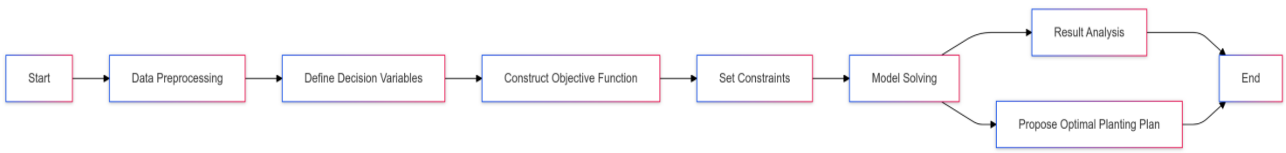

Figure 1. Model Flowchart

Figure 1 illustrates the entire process of the mixed-integer programming model addressing the issue of unsold crops, from start to finish. It includes key steps such as data preprocessing, defining decision variables, constructing the objective function, establishing constraints, solving the model, and analyzing results. Through this process, we can clearly see the establishment and solving of the model, as well as how the optimal planting scheme is ultimately proposed.

3. Results

3.1. Overview of Methods and Problem Solving

3.1.1 Method Application

In response to the situation where the total yield of crops exceeds the expected sales volume, leading to unsold surplus, data visualization analysis was first conducted to gain a more intuitive understanding of the distribution of crop yields and sales conditions. Subsequently, a modeling approach based on mixed-integer programming was adopted. Decision variables were defined (such as those representing planting area, planting decisions, and actual sales volume) to construct an objective function aimed at maximizing planting revenue (where sales revenue is minus planting costs). A series of constraints were established (including area constraints, actual sales volume constraints, crop rotation constraints, legume crop constraints, and minimum planting area constraints), forming a complete mixed-integer programming model. Mathematical optimization algorithms were then applied to solve this model and obtain the optimal planting scheme.

3.1.2 Problem Solving

This method effectively addresses the challenge of how to reasonably allocate the planting area of various crops across different plots in different years under the risk of unsold surplus, thereby maximizing planting revenue. The model comprehensively considers various factors such as the limited availability of land resources, crop growth patterns, market demand uncertainty, and planting costs, providing farmers with a scientific basis for planting decisions. This helps reduce economic losses caused by unsold surplus and improves agricultural production efficiency. By first conducting data visualization analysis, the model construction is more aligned with reality, allowing for more precise decision-making and ensuring the feasibility and effectiveness of the proposed solutions.

3.2. Result Analysis and Phenomenon Explanation

3.2.1 Data Fitting and Visualization

To facilitate data processing, this paper uses the merge function from Pandas to integrate the data from Appendix 1 and Appendix 2, and subsequently visualizes the yields of various crops in 2023, resulting in Figure 2.

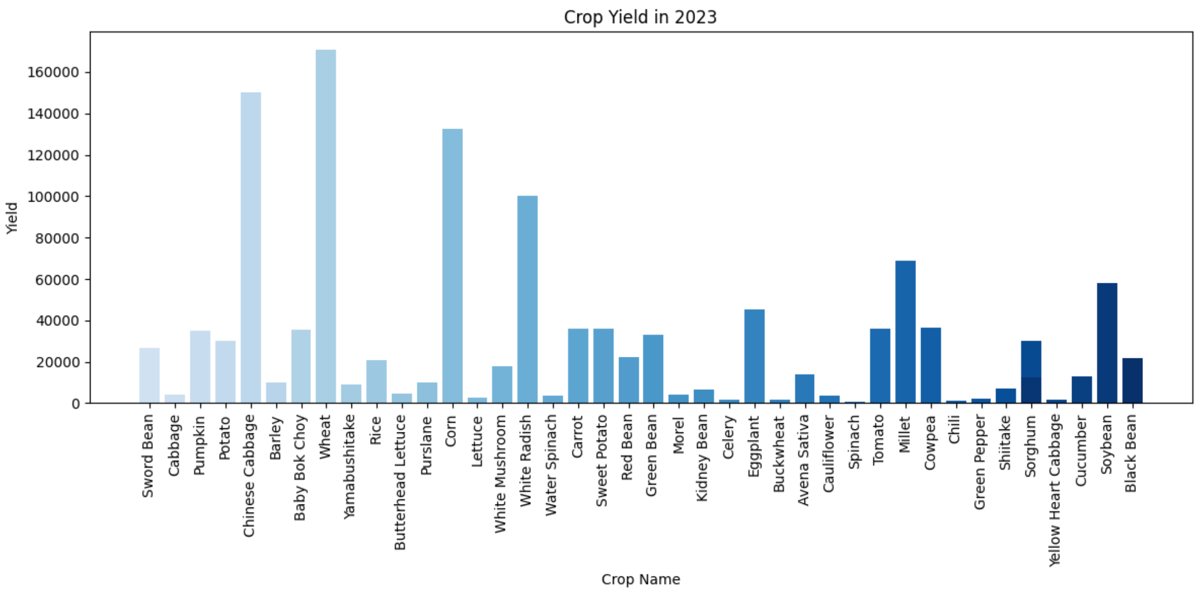

Figure 2. Bar Chart of Crop Yield in 2023

Figure 2 shows the yield of different crops in 2023. It is evident from the chart that there are significant differences in the yields of various crops. Some crops have yields far higher than others, particularly crops like "Wheat," "Corn," and "Chinese Cabbage," which have yields exceeding 120,000 units, indicating their dominant position in the region.

Figure 2 shows the yield of different crops in 2023. It is evident from the chart that there are significant differences in the yields of various crops. Some crops have yields far higher than others, particularly crops like "Wheat," "Corn," and "Chinese Cabbage," which have yields exceeding 120,000 units, indicating their dominant position in the region. The high yields of these crops can be attributed to favorable climatic conditions, fertile and well-drained soil, and the extensive farming experience of the local farmers. Furthermore, the growth in market demand and governmental policy support have also contributed to their leading role. These factors collectively ensure the high production and advantageous status of these crops in the region.

3.2.2 Planting Area Allocation

From the solution results, the allocation of planting areas across different types of plots exhibits certain patterns. The flat dry land, terraced fields, and hilly land primarily grow specific grain crops (excluding rice), with the planting area allocated reasonably based on plot area and the expected yield of various grain crops. For instance, in certain years, the model indicates a larger planting area for sorghum on flat dry land, which may be attributed to the relatively stable market demand for sorghum in the area and the favorable ratio between planting costs and expected returns. This allows sorghum cultivation to contribute significantly to overall revenue while meeting the constraints. Irrigated land is allocated according to planting rules, with part of the area dedicated to rice and part used for growing vegetables in two seasons, fully utilizing the irrigation advantages of these plots and improving land utilization. Ordinary greenhouses strictly adhere to the crop type restrictions for two-season planting, with a reasonable combination of planting areas for first-season vegetables and second-season edible fungi, satisfying market demand for both and ensuring efficient use of greenhouse resources.

3.2.3 The Influence of Planting Cost on Profit

Based on the sensitivity analysis of data perturbation, this paper visually shows the change of annual profit with the year under different planting cost situations, and obtains Figure 3. Figure 3 presents the sensitivity analysis of the planting cost on profit under different planting cost levels. Among them, the overall profit reached its peak in 2024 and 2028, approaching 110 million yuan, while the profit in 2025 and 2029 decreased significantly, with the lowest point approaching 80 million yuan. The influence of various cost levels on profit is relatively consistent, indicating that the change of planting cost will directly affect the profit level. Through the above sensitivity analysis, it is helpful to evaluate the potential impact of different cost strategies on profit.

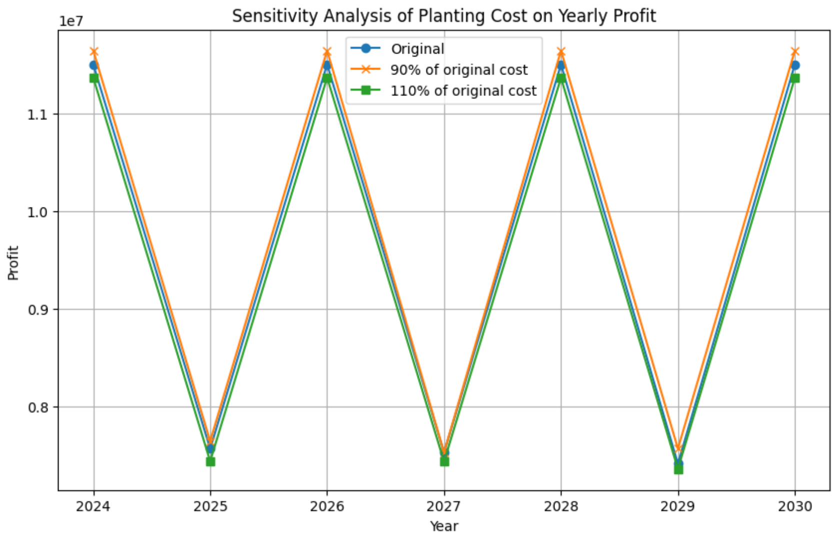

Figure 3. Sensitivity Analysis of Annual Profit to Planting Cost Changes Over Years.

3.2.4 Trend of Profit Changes

Table 1. Maximum Profit Table Under Unsold Conditions

Year	Maximum Profit (CNY)	Profit Change (CNY)	Cumulative Profit (CNY)
2024	11503260.46	—	11503260.46
2025	7571234.11	-3932026.35	19074494.57
2026	11502593.56	3931359.45	30577088.13
2027	7533397.11	-3969196.45	38110485.24
2028	11500748.15	3967351.04	49611233.39
2029	7417775.86	-4082972.29	57029009.25
2030	11502849.46	4085073.6	68531858.71

From Table 1, it can be observed that the profit values from crop cultivation in the village exhibit significant fluctuations across different years. For instance, the maximum profits in 2024, 2026, 2028, and 2030 are relatively close, all around 11.5 million CNY. In contrast, the maximum profits in 2025, 2027, and 2029 are comparatively lower, ranging between 7.4 million and 7.6 million CNY. The profit changes show an alternating pattern of increases and decreases; for example, profits decrease in 2025, increase in 2026, and decrease again in 2027. Overall, despite the occurrence of unsold goods, the business activities continue to accumulate profits.

Analyzing planting revenue year by year reveals that under the model considering unsold products, revenue shows a certain degree of fluctuation. In the early stages, due to uncertainties in market demand, yield, and other factors, revenue may experience some ups and downs. However, as the planting scheme is optimized and adjusted, revenue gradually stabilizes in the later stages. For example, in the first few years of model operation, due to insufficient accuracy in predicting expected market sales and potential impacts from natural disasters affecting yield, some years experienced lower revenue. Nevertheless, as the model continuously adjusts planting areas and crop types based on actual conditions—such as reducing the area for crops with high risks of unsold products and increasing the area for crops with stable and high market demand—revenue gradually stabilizes and shows potential for growth. This indicates that the model possesses dynamic adaptability, enabling it to continuously optimize planting decisions in complex real-world environments to address the challenges posed by unsold products, ultimately achieving or approaching maximum planting revenue.

3.2.5 Impact of Crop Rotation and Legume Constraints

The implementation of crop rotation constraints facilitates reasonable alternation of crops across plots. Results show that the practice of planting the same crop in the same plot for two consecutive years is effectively avoided. This helps maintain soil fertility, reduce pest and disease occurrences, and stabilize crop yields, indirectly affecting planting revenue. For instance, if a plot grows corn in the first year and follows the crop rotation constraint to plant legumes in the second year, this rotation improves soil structure and increases nitrogen content in the soil, creating favorable conditions for subsequent crop growth and reducing planting costs (such as decreased fertilizer usage), thereby enhancing long-term planting revenue. The legume crop constraint ensures that each plot grows at least one legume crop annually, which not only maintains soil ecological balance but also contributes positively to total revenue in years when market demand and prices for legume crops are favorable. For example, in years of high demand and rising prices for soybeans, this constraint ensures soybean cultivation across plots, allowing farmers to seize market opportunities and increase their income.

3.2.6 Actual Sales Volume and Expected Sales Volume Relationship

The actual sales volume constraint ($z_{ijt} \leq EY_{ijt}$ and $z_{ijt} \leq ES_{ijt}$) played a crucial role in the results. During the model's operation, the actual sales volume generally approached or equaled the smaller value between the expected sales volume and the expected yield, effectively preventing inventory backlog and revenue loss caused by overestimating sales volume. This indicates that the model is reasonable in considering the uncertainty of market demand, capable of arranging the planting area according to the expected yield and market expected sales volume, ensuring that crops can be sold on the market in a relatively reasonable quantity, and guaranteeing the stability of planting revenue. For instance, when the expected yield of a certain crop is high but the market expected sales volume is limited, the model will adjust the planting area according to the constraint conditions, ensuring that the actual yield matches market demand, thus avoiding the loss caused by overproduction and unsold crops.

4. Conclusions

This study successfully constructed and applied a mixed-integer programming model to address the issue where the total yield of crops exceeds expected sales volume, leading to unsold surplus. The model optimizes the allocation of planting areas for various crops across different plots and years by reasonably defining decision variables, constructing a scientific objective function, and establishing comprehensive and detailed constraints, thereby maximizing planting revenue. This model provides farmers with a practical and feasible scientific planting decision-making scheme, effectively reducing the backlog of agricultural products and economic losses caused by unsold goods, significantly improving agricultural production efficiency, and holding important practical value for agricultural production decision-making. Furthermore, the findings of this study assist farmers in accurately planning planting schemes, enhancing the market adaptability of agricultural products, increasing economic income, strengthening the ability of agricultural production to cope with market risks, and promoting the robust development of the agricultural industry. Theoretically, this research offers new ideas and methods for optimizing crop planting studies, enriching the research connotation in the field of agricultural economic management, and providing positive references for subsequent related studies.

Although the model constructed in this study takes multiple factors into account, it still has limitations. The model assumes that the selling price of crops is relatively stable, which may not fully align with the volatility of actual market prices. Additionally, the model may not have adequately considered all key factors when simulating the crop growth process, such as pests, diseases, and interactions between crops, which could affect the accuracy of yield predictions. There may also be issues with the accuracy and representativeness of some data due to limitations in understanding the problem or improper data processing during the research process, which could impact the reliability of the model's results. Future research could enhance the model's adaptability by introducing more dynamic price forecasting models that integrate market dynamic data and macroeconomic information for more accurate predictions of crop selling prices. Furthermore, the model could further refine the simulation of the crop growth process, including more precise modeling of the impacts of pests, diseases, and crop interactions, thereby optimizing planting schemes and improving the model's practicality and accuracy, ultimately serving agricultural production decision-making more effectively.

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