Low Volatility Anomaly Analysis Based on the FF-3 and CH-3 Factor Models

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Abstract. Due to its unique investor structure and institutional environment, the Chinese A-share market provides an important context for testing the applicability of classical asset pricing theories. Traditional theories suggest that "high risk corresponds to high returns," but the existence of the low-volatility anomaly (i.e., stocks with low idiosyncratic volatility generate excess returns) challenges this notion. This study aims to verify the universality of the low-volatility anomaly in the Chinese market, providing theoretical support for the improvement of localized asset pricing models and guiding investment practices. Based on the Fama-French three-factor model (FF-3) and the Chinese modified three-factor model (CH-3), this study innovatively combines the EGARCH (1,1) method to dynamically estimate expected idiosyncratic volatility (E(IVOL)) and investigates the relationship between E(IVOL) and returns through univariate and bivariate portfolio analysis. The empirical results show that: (1) there is a significant low-volatility anomaly in the Chinese market, with high E(IVOL) portfolios having lower average monthly returns than low E(IVOL) portfolios. The no-cost strategy's excess returns are 1.24% (t = 5.240) under the FF-3 model and 2.02% (t = 7.048) under the CH-3 model; (2) the CH-3 model, by eliminating shell value interference and optimizing the value factor, captures the anomaly significantly better than the FF-3 model, highlighting the practical value of localized models. This research not only provides new evidence on the cross-country differences of the low-volatility anomaly but also offers methodological references for asset pricing and portfolio management in the Chinese market.

Keywords: Low idiosyncratic volatility anomaly, CH-3 factor, EGARCH, Portfolio analysis.

1. Introduction

Return and risk are central topics in financial research. Classical pricing models generally suggest a positive relationship between expected return and risk, such that higher risk should be associated with higher expected return[1]. However, Fama and French demonstrated that stocks with low β (beta) tend to yield higher returns, while stocks with high β tend to have lower returns. This phenomenon, known as the low β anomaly or the low-risk anomaly[2], challenges the assumptions of traditional pricing models.

Traditional pricing models generally account solely for systematic risk, excluding non-systematic risk, which can be entirely diversified away. Non-systematic risk is also known as idiosyncratic volatility. Ang et al. (2006) [3] examined 23 developed markets and discovered that stocks exhibiting high idiosyncratic volatility yielded lower future average returns, whereas those with low idiosyncratic volatility produced greater average future returns. This discovery challenges the conventional financial idea that "greater risk results in greater return." The occurrence of low idiosyncratic volatility equities yielding superior future returns is termed the low idiosyncratic volatility anomaly. Investigations into this anomaly generally utilize the residual standard deviation derived from pricing models as an indicator of idiosyncratic volatility. Hsu and Chen (2017)[4] employed the residual standard deviation derived from the Fama-French three-factor model to quantify idiosyncratic volatility. Fu (2009) [5] utilized the EGARCH model to assess idiosyncratic volatility, with the mean equation derived from the Fama-French three-factor model. Jianan Liu et al. (2019)[6] introduced the CH-3 model, highlighting its superior performance compared to the FF-3 model within the Chinese environment.

The study aims to investigate the "low-volatility anomaly" in China's A-share market, specifically to examine whether systematic pricing errors occur concerning idiosyncratic volatility. This study employs the CH-3 and FF-3 factor models, using the EGARCH model to estimate expected idiosyncratic volatility under both models. By constructing investment portfolios based on the differences in expected idiosyncratic volatility, this paper demonstrates that these portfolios can generate excess returns, confirming the negative correlation between non-systematic risk and return, and validating the existence of the "low-volatility anomaly" in the Chinese market.

2. Methodology

2.1. Estimating Expected Idiosyncratic Volatility Based on the CH-3 Model

The CH-3 model is characterized by two key features. First, when constructing the market capitalization factor, the bottom 30% of companies by market capitalization are excluded, as these small-cap companies may possess shell value[6]. Second, the value factor is based on the price-to-earnings ratio.

The remaining 70% of stocks are categorized into two groups: small-cap (S) and large-cap (B), with allocation based on the median market price. These groups are further divided into three EP categories: the top 30% (value V), the middle 40% (neutral M), and the bottom 30% (growth G). The intersections of these groups create six "size-EP" combinations, which are then used to form value-weighted investment portfolios: S/V, S/M, S/G, B/V, B/M, and B/G. When constructing value-weighted portfolios, each stock is weighted by its market capitalization in all circulating A-shares (including non-tradable shares). The size and value factors in this model are represented as SMB (small minus big) and VMG (value minus growth), and the returns of the six portfolios are combined as follows:

$$\begin{aligned} SMB &= \frac{1}{3}\left(\frac{S}{V}+\frac{S}{M}+\frac{S}{G}\right)-\frac{1}{3}\left(\frac{B}{V}+\frac{B}{M}+\frac{B}{G}\right) \\ VM &= \frac{1}{2}\left(\frac{S}{V}+\frac{B}{V}\right)-\frac{1}{2}\left(\frac{S}{G}+\frac{B}{G}\right) \end{aligned} \tag{1}$$

The CH-3 model is used to estimate the expected idiosyncratic volatility (IVOL) of individual stocks[6]. The specific model is as follows:

$$R_{i,t}-r_{f,t}=\alpha_{f,t}+\beta_{MKT}\left(R_{m,t}-r_{f,t}\right)+\beta_{SMB}SMB_t+\beta_{VMG}VMG_t+e_{i,t} \tag{2}$$

The GARCH model is widely employed for modeling conditional return volatility. This study utilizes the Exponential GARCH (EGARCH) model to estimate the conditional variance, denoted as σ^2.[7] The model depends on the past variance over p periods and the mean-reversion over q periods:

$$\ln\sigma_{it}^2=a_i+\sum_{l=1}^{p}b_{i,l}\ln\sigma_{i,t-l}^2+\sum_{k=1}^{q}C_{i,k}\{\theta(\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}})+\gamma[|\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}}|-(2/\pi)^{\frac{1}{2}}]\} \tag{3}$$

The expected idiosyncratic volatility (E(IVOL)) is computed as the square root of the conditional variance. The EGARCH (1,1) model is applied to extract IVOL, guided by the AIC criterion for minimizing information loss. In this model, a negative γ indicates a negative correlation between volatility and return.

2.2. Estimating Expected Idiosyncratic Volatility Based on the FF-3 Model

The second method employed in this paper to estimate E(IVOL) is based on the approach used by Fu (2009)[5], which extracts IVOL using the FF-3 model in conjunction with the EGARCH(p,q) model. The exact specification of the FF-3 model is provided below:

$$R_{i,t}-r_{f,t}=\alpha_{i,t}+\beta_i\left(R_{m,t}-r_{f,t}\right)+s_iSMB_t+h_iHML_t+e_{i,t} \tag{4}$$

The research hypothesis proposed is that expected idiosyncratic volatility (IVOL) is negatively correlated with stock returns. To test this relationship between IVOL and stock returns, it is essential to accurately estimate IVOL, as expected idiosyncratic risk is not directly observable. The conventional approach involves using the error standard deviation from the FF-3 model as a proxy for IVOL (the explanatory variable) in cross-sectional regressions. Given the need for accurate results and the specific context of the Chinese market, this paper employs both the CH-3 and FF-3 models, with the assumption that the error terms follow the EGARCH(p,q) model. The expected idiosyncratic volatility series for individual stocks are estimated within and out-of-sample, which will be used for further analysis.

$$R_i = \gamma_{ot} + \gamma_{tt} E_{t-l}[\text{ IVOLLit }_{\text{it}}] + \sum_{k=2}^{K} \gamma_{kt} E_{t-l}[\text{X}_{\text{kit}}] + \varepsilon_{it} \quad i = 1,2,\dots,N_t, t = 1,2,\dots,T \tag{5}$$

Portfolio analysis methods are widely used in the literature due to their ease of implementation and the fact that they do not require assuming a linear relationship between expected returns and risk factors[8]. Additionally, the difference in expected returns between the highest and lowest portfolios can be interpreted as the profit from a trading strategy. From the perspective of E(IVOL), stocks are ranked based on their E(IVOL) values, divided into five portfolios, and held for a specified period. The return difference between the portfolios with the highest and lowest E(IVOL) values is then analyzed to determine if the difference is statistically significant. If the portfolio with the highest E(IVOL) shows significantly lower returns than the portfolio with the lowest E(IVOL), it would provide evidence of the low idiosyncratic volatility anomaly in the Chinese stock market. The process of portfolio analysis is as follows: every month, stocks are ranked according to their E(IVOL) and divided into five portfolios, each containing an equal number of stocks. The first portfolio includes the 20% of stocks with the lowest E(IVOL), while the last portfolio contains the 20% of stocks with the highest E(IVOL).

The bivariate portfolio analysis method primarily investigates whether other factors influence the relationship between IVOL and stock returns[9]. Using company size as an example, stocks are first sorted by market capitalization to create quintile portfolios for controlling size. Then, within each size quintile, stocks are ranked based on their E(IVOL) and divided into five portfolios. This process is repeated every month. Portfolio P1 includes stocks with the lowest E(IVOL) and smallest company size, while portfolio P25 contains stocks with the highest E(IVOL) and largest company size. The average return of each portfolio is then calculated.

Table 1 defines the names, symbols, and specific meanings of various variables used in this study.

Table 1. Variable Definitions

Variable Name	Variable Symbol	Variable Definition
Return	Ri	The realized return of stock i
Book-to-Market Ratio	B/M	The ratio of the book value to the market value of the company
Market Capitalization	Size	The natural logarithm of the product of outstanding shares and stock price
Characteristic Volatility (FF3)	E(IVOL)-FF3	The EGARCH (1,1) time series of the residuals from the Fama-French 3-factor model
Characteristic Volatility (CH3)	E(IVOL)-CH3	The EGARCH (1,1) time series of the residuals from the China 3-factor model
Turnover Rate	Turnover	The ratio of trading volume to the outstanding shares
Leverage	Leverage	The debt-to-equity ratio: total liabilities divided by total assets
Market Portfolio	MKT	The value-weighted return of the market portfolio
CH-3 Size Factor	CHSMB	The difference in returns between small-cap and large-cap stock portfolios (excluding the smallest 30% by market cap)
CH-3 Value Factor	VMG	The difference in returns between high-profitability market-to-book ratio stocks and low-profitability market-to-book ratio stocks
FF-3 Size Factor	FFSMB	The difference in returns between small-cap and large-cap stock portfolios
FF-3 Value Factor	FFHML	The difference in returns between high book-to-market ratio stocks and low book-to-market ratio stocks

3. Empirical Analysis

3.1. Data Sources and Preprocessing

Stock lists are first obtained from the CSI 300 Index (000300.XSHG) and the SME Board Index (399107.XSHE). A series of selection criteria are applied as follows:

(1) Exclude the lowest 30% of stocks by market capitalization

(2) Exclude stocks listed for less than 60 months

(3) Exclude stocks with fewer than 120 trading days in a year or fewer than 15 trading days in a month.

The time window spans from January 2004 to December 2023, with a total of 240 monthly return data points.This study excludes stocks that were publicly listed in the past 60 months. To mitigate the impact of outliers on the research results, all continuous variables are winsorized at the 1% level. Since the empirical analysis follows the Chinese version of the three-factor model (CH3), which modifies the Fama-French three-factor model, necessary screening of the stock pool is conducted. According to Liu et al. (2019)[7], the smallest 30% of listed companies in the Chinese stock market are severely affected by shell-value contamination, which causes asset pricing models to fail in accurately reflecting the differences in expected stock returns across the cross-section. To better study the pricing mechanism of A-shares, this paper filters out the smallest 30% of companies by market capitalization. After the data screening and processing, the final dataset includes 2,507 companies.

3.2. Descriptive Statistic analysis

Table 2 provides descriptive statistics of the sample data. The sample of 2,507 companies has an average monthly return of 0.80% over the entire period. For the FF-3 and CH-3 models, the monthly averages of E(IVOL) calculated by the two methods are 12.10% and 9.92%, respectively. The average logarithm of market capitalization is 14.73. The average book-to-market ratio is 0.435, meaning that the book value is, on average, 0.435 times the market value. On average, the companies exhibit relatively low leverage, with an average leverage ratio of 0.362. The average turnover rate, a measure of liquidity, is 0.86.

Table 2. Descriptive Statistics of the Sample Data

Variable	Symbol	Mean	Std Dev	1%	25%	50%	75%	99%
Return	R	0.80%	20.95%	-32.10%	-7.30%	0.01%	8.20%	45.60%
B/M	BM	0.435	0.255	0.003	0.192	0.323	0.483	1.237
Size	ln(ME)	14.73	1.08	11.45	12.73	13.32	15.12	17.89
E(IVOL)-FF3	σ_FF3	12.10%	5.84%	5.62%	7.32%	9.17%	11.20%	21.80%
E(IVOL)-CH3	σ_CH3	9.92%	4.28%	5.11%	7.04%	8.99%	10.80%	20.10%
Turnover	TO	0.86	0.48	0.07	0.42	0.85	1.64	5.74
Leverage	Lev	0.362	0.29	0.017	0.254	0.325	0.397	0.462

3.3. Empirical Analysis through Portfolio Construction

3.3.1. Univariate Portfolio

From 2004 to the end of 2023, this study sorts all sample stocks on a monthly basis according to the IVOL calculated using the CH-3/FF-3 model and EGARCH(1,1) model. Table 3 shows that the average returns of portfolios formed based on E(IVOL) decrease from 0.62% per month for Q1 (stocks with lower E(IVOL)) to -1.53% per month for Q5 (stocks with higher E(IVOL)). In the second row, the FF-3 alpha value for Q5 is -0.76% per month. The difference in FF-3 alpha values between Q1 and Q5 (a zero-cost portfolio, short Q5, long Q1) is 1.31% per month, with a robust t-statistic of 3.325, indicating that the result is statistically significant.

Table 3. Portfolio Return Analysis Formed Based on FF3-E(IVOL)

	Q1	Q2	Q3	Q4	Q5	1-5
Average Return	0.62	-0.23	-0.66	-0.47	-1.53	2.15*** [3.989]
Alpha (FF3)	0.55* [1.724]	-0.13 [-0.232]	-0.32 [-0.782]	-0.25 [-1.141]	-0.76** [-2.147]	1.31*** [3.325]

The empirical results in Table 4 indicate that portfolios of stocks with lower E(IVOL) have higher returns, while those with higher E(IVOL) display lower returns, consistent with the 'low volatility anomaly'. In the second row, the CH-3 alpha value for Portfolio 5 is -0.78% per month, which is statistically significant at the 5% level. The difference in CH-3 alpha between Portfolio 1 and Portfolio 5 (a zero-cost portfolio, short Q5, long Q1) is 2.15% per month, with a robust t-statistic of 3.227. The return analysis of portfolios formed based on CH-3 E(IVOL) supports the 'low volatility anomaly', showing a significant negative correlation between E(IVOL) and stock returns.

Table 4. Portfolio Return Analysis Formed Based on CH3-E(IVOL)

	Q1	Q2	Q3	Q4	Q5	1-5
Average Return	1.17	0.58	0.37	0.22	-0.49	1.66*** [8.112]
Alpha (CH3)	1.37** [2.462]	0.33 [0.869]	-0.32 [-0.782]	-0.28 [-1.254]	-0.78** [-2.025]	2.15*** [3.227]

3.3.2. Bivariate Portfolio

This study attempts to examine whether the positive correlation between E(IVOL) and stock returns persists after controlling for other company characteristics. First, at the end of each month, stocks are categorized into five portfolios based on company characteristics (such as Beta value, company size, market-to-book ratio, turnover rate, and liquidity), and within each portfolio, stocks are further sorted according to E(IVOL).

(1) FF-3 Model Test Results

Table 5 presents the results of the bivariate portfolio return analysis based on the FF-3 model. Companies with high Beta values bear greater systematic risk and thus receive relatively higher compensation. After accounting for this effect, the average return difference between low E(IVOL) and high E(IVOL) portfolios is 1.70%, with an FF-3 alpha value of 1.24% and a Newey-West t-statistic of 5.240. Therefore, Beta cannot fully explain the E(IVOL) premium.

After controlling for company size, the average return difference between low E(IVOL) and high E(IVOL) portfolios is 0.89%, with an FF-3 alpha value of 0.67% and a Newey-West t-statistic of 2.812. Thus, market capitalization cannot explain the high (low) returns of high (low) expected E(IVOL) stocks.

The average realized return difference between the high and low E(IVOL) quintile portfolios is 1.71% per month, with the FF-3 alpha difference between portfolios 5 and 1 also being positive, at 0.93% per month, and statistically significant. The corresponding Newey-West t-statistic is 2.594.

After controlling for turnover ratio, it can be observed that turnover ratio cannot explain the IVOL premium. The average return of the zero-cost portfolio formed based on E(IVOL) is 2.35% per month. Its FF-3 alpha is also positive and highly significant, at 1.34% per month. The Newey-West t-statistic is 5.410.

Table 5. Bivariate Portfolio Return Analysis Based on FF3-E(IVOL)

Portfolio Formed Based on FF3-E(IVOL)							
Control Variables	Average Return						(5-1) Alpha(FF-3)
	Min	2	3	4	Max	5-1	
Beta Value	-0.93	-0.63	-0.32	0.01	0.77	1.70	1.24* [5.240]
Size	0.06	0.34	0.54	0.75	0.95	0.89	0.67** [2.812]
Book-to-Market Ratio	-0.68	0.61	0.39	0.55	1.03	1.71	0.93** [2.594]
Turnover Rate	0.93	0.27	0.12	0.07	0.01	0.92	0.74** [5.410]
Leverage	-1.31	0.93	0.74	0.32	0.01	1.32	1.10** [3.684]

Under the condition that asset volatility and expected asset returns remain unchanged, increasing leverage raises expected stock returns. Asset volatility also prevents firms from increasing leverage. Therefore, companies with higher IVOL may have higher asset volatility, but due to lower leverage, their equity returns are relatively lower. After controlling for this effect, the average realized return difference between high and low E(IVOL) quintile portfolios is 1.32% per month. The FF-3 alpha difference between portfolios 5 and 1 is 1.10% per month and is highly significant. The corresponding Newey-West t-statistic is 3.684.

(2) CH-3 Model Test Results

Table 6 presents the results of the bivariate portfolio return analysis based on the CH-3 model. Under the CH-3 model, after controlling for Beta, the average return difference between the low E(IVOL) and high E(IVOL) portfolios is 1.79%, with the CH-3 alpha value of 2.02% and the Newey-West t-statistic of 7.048. After controlling for firm size, the average return difference between the low E(IVOL) and high E(IVOL) portfolios is 0.92%, with the CH-3 alpha value of 1.23% and the Newey-West t-statistic of 6.137. Therefore, market capitalization cannot explain the high (low) returns of high (low) E(IVOL) stocks.

In the third row, after controlling for the Book-to-Market ratio (BM), the average realized return difference between the high and low E(IVOL) quintile portfolios is 1.89% per month. The CH-3 alpha difference between portfolios 5 and 1 is also positive, at 0.92% per month, and statistically significant. The corresponding Newey-West t-statistic is 4.105. In the fourth row, after controlling for turnover ratio (representing liquidity), the relationship between E(IVOL) and stock returns is shown. The average return of the zero-cost portfolio formed based on expected idiosyncratic volatility is 0.77% per month. Its CH-3 alpha is also positive and highly significant, at 1.52% per month. The Newey-West t-statistic is 8.096. In the fifth row, after controlling for leverage, the average realized return difference of the zero-cost portfolio is 3.01% per month. The CH-3 alpha difference between portfolios 5 and 1 is also positive, at 2.05% per month, and highly significant. The corresponding Newey-West t-statistic is 9.043.

Table 6. Bivariate Portfolio Return Analysis Based on CH3-E(IVOL)

Portfolio Formed Based on CH3-E(IVOL)							
Control Variables	Average Return						(5-1) Alpha (CH-3)
	Min	2	3	4	Max	5-1	
Beta Value	-0.97	-0.68	-0.43	0.02	0.87	1.79	2.02*** [7.048]
Size	0.08	0.39	0.52	0.63	1.02	0.92	1.23** [6.137]
Book-to-Market Ratio	-0.72	0.52	0.61	0.71	1.21	1.89	0.92*** [4.105]
Turnover Rate	0.82	0.31	0.21	0.11	0.04	0.77	1.52*** [8.096]
Leverage	-1.49	-0.58	0.62	0.72	1.48	3.01	2.05** [9.043]

Investors who hold a zero-cost portfolio formed according to E(IVOL)-FF3 (shorting the highest E(IVOL) stock portfolio and going long the lowest E(IVOL) stock portfolio) would, on average, achieve an abnormal monthly return of 1.70%. However, for a zero-cost portfolio formed according to E(IVOL)-CH3, the average abnormal monthly return is 1.79%. Both the FF3 and CH3 models reveal statistically significant alpha values in the trading strategies. In comparison, the portfolio formed according to E(IVOL)-CH3 yields higher abnormal returns. The bivariate portfolio analysis also shows similar results. The empirical findings suggest the presence of the low idiosyncratic volatility anomaly in the Chinese market.

4. Conclusion

This study aims to validate the existence of the "low volatility anomaly" in the Chinese A-share market, specifically the negative correlation between idiosyncratic volatility and stock returns. Based on the classic FF-3 model and the China-specific CH-3 model, this paper innovatively combines the EGARCH(1,1) method to estimate the expected idiosyncratic volatility (E(IVOL)) and investigates its relationship with returns through both univariate and bivariate portfolio analyses. The empirical results show that:

(1) Whether using the FF-3 or CH-3 model, the returns of the high E(IVOL) portfolio are significantly lower than those of the low E(IVOL) portfolio, confirming the existence of systematic pricing biases in the Chinese market and supporting the universality of the "low volatility anomaly."

(2) The CH-3 model, by excluding the interference of small-cap stocks and optimizing the value factor, demonstrates significantly superior anomaly-capturing ability compared to the FF-3 model. The constructed zero-cost portfolio achieves an average monthly excess return of 1.79% (t = 7.048), highlighting the applicability of localized models in the Chinese market.

Future research could further explore the following areas based on the model:

(1) Model Optimization: Further examine the robustness of the CH-3 model across different market cycles, such as the period of extreme volatility in the stock market in 2015, and attempt to integrate dynamic factors (e.g., liquidity factors) to enhance explanatory power[10].

(2) Expansion of Research Scope: Extend the sample to include companies from the STAR Market or the Beijing Stock Exchange, test the anomaly's performance in emerging sectors, and explore its relationship with corporate governance and the quality of information disclosure.

(3) Multidimensional Risk Control: Incorporate additional risk-adjusted metrics (such as maximum drawdown and tail risk[11]) in portfolio construction to evaluate the practical applicability of the strategy.

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