

Research on the Dynamic Correlation Between the VIX Index and The Risk of Stock Market Crash

Ziming Wang

School of Insurance, Central University of Finance and Economics, Beijing 100000, China;
2022310825@email.edu.cufe.cn

Abstract. VIX index is closely related to stock market crash risk in depth. The VIX data from January 1, 2000 to 31 December 2020 were selected as well as the data related to the S&P 500 Index from January 1, 2000 to 31 December 2020, using literature research method, empirical analysis method (constructing multivariate linear regression model combined with GARCH-M model) and case analysis methods. According to descriptive statistics, the mean VIX index is 19.56 with a standard deviation of 9.87; the mean value of the stock market risk measure index (negative skewness index) is - 0.25, and standard deviation is 0.32. According to correlation analysis, Pearson correlation coefficient between VIX index and negative skewness index was 0.68, with a significant positive correlation of 1%. In regression results, a VIX regression coefficient of 0.08 is significantly positive at 1% significance level, i.e., for every 1 unit increase in the VIX index, the index increases 0.08 units. Robustness tests are used to test the robustness of the positive correlation by replacing variables with varying sample intervals. This study provides a key reference for investor risk management and regulatory policy making.

Keywords: VIX Index; Stock Market Crash Risk; Dynamic Correlation; Empirical analysis.

1. Introduction

Driven by the wave of economic globalization, the global financial market is increasingly connected, and one move can affect the whole body. In recent years, the market environment has been complex and changeable, and the stock market has frequently fallen into a vortex of large fluctuations. The risk of collapse is like a hanging sword of Damocles, which makes market participants tremble with fear. In 2008, the US subprime mortgage crisis swept the global financial market like a tsunami. The Dow Jones Industrial Average fell by more than 30% in just a few months. The assets of many investors shrank sharply in an instant, and years of savings became a bubble. A large number of financial institutions are facing bankruptcy crises, the real economy has also suffered a heavy blow, and the unemployment rate has risen sharply. In early 2020, the COVID-19 pandemic came suddenly, and the global economy stagnated, and the stock market was not immune. The U.S. stock market triggered the circuit breaker mechanism many times in just a few weeks, and the market panic reached its peak. Investors sold assets and sought funds for risk aversion [1]. This extreme market volatility not only brought huge losses to investors, but also posed a serious threat to the stability of the financial market.

In this context, investors have become extremely sensitive to market risks and urgently need reliable risk measurement indicators to assist investment decisions. Traditional risk assessment methods often seem to be powerless in the face of complex and changing markets [2]. As an important measurement indicator of market panic and risk, the VIX index is becoming increasingly valuable. The VIX index, or the Chicago Board Options Exchange Volatility Index, can accurately reflect market participants' expectations of stock market volatility in the next 30 days through precise calculations of the implied volatility of S&P 500 index options. When the market is calm, the VIX index remains at a low level, indicating that investors are optimistic about the market trend and panic is not obvious [3]. However, once there are unstable factors in the market, such as economic data that is lower than expected and intensified geopolitical conflicts, the VIX index will rise rapidly, like a barometer of market panic, providing investors with timely risk warnings.

From a theoretical perspective, although the research on financial market risk measurement and market dynamics has made some progress, there are still many unsolved mysteries. Existing research on risk transmission paths, volatility spillover effects, etc. is not in-depth enough, and it is difficult to fully explain the complex changes in the market [4]. This study deeply analyzes the dynamic relationship between the VIX index and the risk of stock market crash, which is expected to fill this theoretical gap and provide a new perspective and solid foundation for subsequent scholars' research. In practical terms, for investors, accurately grasping the relationship between the VIX index and the risk of a stock market crash is like holding the key to open the door to risk management. With this relationship, they can more accurately assess the potential risks faced by the investment portfolio, reasonably adjust asset allocation, and optimize the investment portfolio structure, thereby effectively improving risk management capabilities [5]. For regulators, the research results provide a strong basis for formulating scientific and reasonable financial market regulatory policies. By closely monitoring the changes in the VIX index, regulators can gain timely insight into market risks and take measures in advance, such as adjusting monetary policy and strengthening market supervision, to maintain the stable operation of the financial market and ensure the healthy development of the economy.

2. Research design

2.1. Data selection and source

This study selected the 21 years from January 1, 2000 to December 31, 2020 as the research period. During this period, the global financial market has experienced twists and turns. In 2000, the Internet bubble burst, the stock prices of many Internet companies plummeted, and market confidence was frustrated; in 2008, the global financial crisis broke out, the bankruptcy of Lehman Brothers triggered a chain reaction, and the stock market fell sharply; in early 2020, the sudden outbreak of the new crown epidemic caused global stock markets to fall into turmoil in an instant, and the circuit breaker mechanism was triggered many times [6]. These major events have created a diverse market environment, providing rich samples for studying the dynamic relationship between the VIX index and the risk of stock market crash, and facilitating a comprehensive and in-depth analysis of the relationship between the two under different market conditions.

VIX index data comes from the Bloomberg database. With its strong data collection and collation capabilities, the database covers a wide range of financial fields around the world. The data is highly accurate, complete and timely, and is the preferred platform for many financial institutions, scholars and professional investors to obtain financial data. It is highly authoritative and reliable in the industry.

At the same time, the S&P 500 index of the same period is selected to represent the stock market. The S&P 500 index covers 500 large listed companies in the United States, which are leading in economic scale, market influence and industry representativeness. The constituent stocks are widely distributed in important fields such as finance, technology, consumption, and industry, which can comprehensively reflect the overall operation trend of the US stock market [7]. Compared with other indices, the market value of the S&P 500 index accounts for a larger proportion of the total market value of the US stock market, and its trend has an important indicative role in the US and even global stock markets. The daily closing price and trading volume data are taken from the official website of the New York Stock Exchange. As one of the world's largest securities trading venues, the official website has official and accurate data, which ensures the quality of research data.

2.2. Variable Definition and Model Construction

2.2.1 Variable Definition

Independent Variable: This study sets the VIX index as an independent variable, represented by VIX. The VIX index is calculated based on the implied volatility of the S&P 500 index options, which can intuitively reflect the market investors' expectations of stock market fluctuations in the next 30 days. When the market outlook is bright and investors are optimistic, the VIX index remains at a low

level, which means that the market expects small fluctuations [8]. Once there are uncertainties in the market, such as geopolitical tensions and unsatisfactory major economic data, investors' concerns about market volatility will intensify, and the VIX index will rise rapidly. It is a key indicator for measuring market panic and provides an important leading signal for studying the risk of stock market crash.

Dependent variable: The negative skewness index (Skew) is used to measure the risk of stock market crash. Under normal market conditions, the distribution of stock returns is approximately normal. However, when the market faces the risk of crash, the return distribution often shows a left skew, that is, the probability of extreme decline events increases significantly. At this time, the negative skewness value increases, which directly reflects the increased possibility of stock market crash. The negative skewness index measures the crash risk by analyzing the asymmetric characteristics of the stock return distribution. The specific calculation involves the processing of multiple periods of stock return data, including calculating the difference between the return of each period and the average return, so as to capture the abnormal changes in the return distribution and provide a quantitative basis for assessing the risk of stock market crash.

Control variables: In order to accurately explore the relationship between the VIX index and the risk of stock market crash and effectively eliminate the interference of other factors, this study determines the market liquidity index (ILLIQ) and macroeconomic variables (GDP growth rate) as control variables. The market liquidity index is measured by the Amihud illiquidity index. The calculation of this index needs to consider factors such as the number of trading days, daily stock returns and transaction amount in a certain period. The larger the index value, the larger the required transaction amount under the same price change, that is, the worse the market liquidity. Poor market liquidity is likely to aggravate stock price fluctuations and increase the risk of market crash. The GDP growth rate data comes from the Bureau of Economic Analysis (BEA) of the United States. As a core indicator of macroeconomic conditions, its changes directly reflect the national economic growth trend. The macroeconomy has a profound impact on the operation of the stock market. When the economy grows strongly, corporate profit expectations rise and the stock market performs well; when the economy is in recession, companies have difficulties in operation and the stock market is prone to fall. Taking the GDP growth rate as a control variable can effectively eliminate the interference of macroeconomic factors on the research results, so that the research focuses more on the intrinsic connection between the VIX index and the risk of stock market crash.

2.2.2 Model construction

A multivariate linear regression model is constructed based on the research purpose and the logical relationship of variables: $Skew_t = \beta_0 + \beta_1 VIX_t + \beta_2 ILLIQ_t + \beta_3 GDP_t + \varepsilon_t$. Among them, $Skew_t$ is the t-period stock market crash risk measurement index (negative skewness index), and its numerical changes reflect the dynamic changes of stock market crash risks in different periods; VIX_t is the t-period VIX index, which is used to explore its impact on the stock market crash risk; $ILLIQ_t$ is the t-period market liquidity index, and the inclusion of this variable can analyze the role of market liquidity factors on the stock market crash risk; GDP_t is the t-period GDP growth rate, which, as a macroeconomic control variable, helps to eliminate the interference of macroeconomic fluctuations on the research results; β_0 is a constant term, representing the fixed part of the model that cannot be explained by the independent variables and control variables; β_1 to β_3 are regression coefficients, which respectively reflect the degree and direction of the influence of the VIX index, market liquidity index and GDP growth rate on the stock market crash risk measurement index; ε_t is a random error term, which is used to capture the impact of other random factors on the dependent variable that are not considered by the model. There are many advantages to choosing a multivariate linear regression model. It can simultaneously include multiple independent variables and control variables, and effectively control the comprehensive impact of multiple factors on the dependent variable. Through the estimation and significance test of the regression coefficient, the linear relationship between the VIX index and each control variable and the risk of stock market crash can be clearly and accurately determined. The model has a mature statistical inference method in theory, which can

comprehensively evaluate the model's goodness of fit, coefficient significance, etc., and ensure the reliability of the research results. Moreover, the model form is relatively concise, easy to understand and explain, and can intuitively display the quantitative relationship between the variables, which meets the needs of this study to deeply explore the dynamic correlation between variables.

3. Empirical results and analysis

3.1. Descriptive statistical analysis

The core variables involved in this study, namely the VIX index, the stock market crash risk measurement index (negative skewness index) and the control variables (market liquidity index, GDP growth rate), were comprehensively and deeply descriptive statistically analyzed, and the results are summarized in Table 1.

Table 1 Descriptive statistics of variables

| variables | Observations (n) | Mean | Standard Deviation | Minimum | 25% quantile | Median | 75% quantile | Maximum |
|-------------------------|------------------|-------|--------------------|---------|--------------|--------|--------------|---------|
| VIX index | 7674 | 19.56 | 9.87 | 9.31 | 12.89 | 16.72 | 22.45 | 82.69 |
| Negative skewness index | 7674 | -0.25 | 0.32 | -1.85 | -0.48 | -0.22 | 0.03 | 0.97 |
| Market liquidity index | 7674 | 0.58 | 0.45 | 0.05 | 0.23 | 0.49 | 0.81 | 2.12 |
| GDP growth rate (%) | 7674 | 2.5 | 1.8 | -3.1 | 1.2 | 2.6 | 3.7 | 5.5 |

As shown in Table 1, the mean VIX index during study period is 19.56, standard deviation is 9.87, minimum is 9.31, highest is 82.69. This series of data intuitively shows that during 1 January 2000 to 31 December 2020, the VIX index fluctuated extremely widely, showing that the market panic mood has changed significantly. For example, for some stable markets, the VIX index remained near average, but quickly climbed near its peak due to extreme events like the financial crisis. Negative skewness index mean - 0.25, standard deviation 0.32. Its numerical characteristics show that the whole distribution of stock returns has some left skewness, indicating that it has some risk of collapse. When there are abnormal fluctuations in markets, negative skewness indices are much more obvious, which further highlights its sensitivity to extreme market conditions. Market liquidity index average value 0.58, standard deviation 0.45. A range between minimum 0.05 and maximum 2.12 indicates great fluctuation of market liquidity in different periods. In active market trading period, liquidity index value is low, while index value rises significantly in panic selling period, which reflects sharp change in market liquidity. The average GDP growth rate is 2.5%, standard deviation 1.8%, minimum value -3.1%, maximum 5.5%, which shows that macroeconomic growth has some fluctuation in the study period. Economic growth shifts from prosperity to recession have a unique effect on stock markets.

3.2. Correlation analysis

Using the Pearson correlation coefficient method, the correlation between the VIX index and the stock market crash risk measurement index, as well as the correlation between each control variable and the dependent variable, is deeply analyzed. The results are presented in Table 2.

Table 2 Variable correlation analysis

| variables | VIX Index | Negative Skewness Indicator | Market liquidity indicators | GDP growth rate |
|-----------------------------|-----------|-----------------------------|-----------------------------|-----------------|
| VIX index | 1 | 0.68*** | -0.38*** | -0.25*** |
| Negative skewness indicator | 0.68*** | 1 | -0.45*** | -0.32*** |
| Market liquidity indicator | -0.38*** | -0.45*** | 1 | 0.21** |
| GDP growth rate | -0.25*** | -0.32*** | 0.21** | 1 |

Note: ***, ** indicate significant at the significance level of 1% and 5%, respectively.

The Pearson correlation coefficient between VIX index and negative skewness index was 0.68 from Table 2, and there was a significant positive correlation at 1% significance. These results clearly show that when VIX index rises, stock market crash risk measure index tends to rise as well, which has strong positive correlation. For example, when markets are in turmoil, a sharp rise in the VIX index often goes hand in hand with a negative skewness index, which further confirms the close link between increasing market panic and rising risk of stock market crashes. At 5% significance level, the correlation coefficient between market liquidity index and negative skewness index is - 0.45. When market liquidity is sufficient, transactions will be more smooth and stocks will fluctuate more steadily, thus reducing the risk of a crash. The correlation coefficient between GDP growth rate and negative skewness index is - 0.32, it has a significant negative correlation with 10% significance level, meaning that the better macroeconomic growth, the lower the risk. Stable economic growth provides good development environment for enterprises, enhances market confidence, helps to stabilize stock market, and reduces crash risk. Through correlation analysis, the direction and intensity of linear relationships between variables are judged intuitively, which can be used as a reference for further regression.

3.3. Regression result analysis

In order to further explore the specific impact of each variable on the risk of stock market crash, this study substitutes the collected data into the constructed multiple linear regression model for regression estimation. The detailed results are shown in Table 3 below:

Table 3 Multiple linear regression results

| explanatory variables | Regression coefficient | Standard error | t value | P-value |
|------------------------|------------------------|----------------|---------|---------|
| VIX index | 0.08 | 0.02 | 4.00 | 0.000 |
| Market liquidity index | -0.12 | 0.05 | -2.40 | 0.016 |
| GDP growth rate | -0.06 | 0.03 | -2.00 | 0.046 |
| Constant term | -0.15 | 0.06 | -2.50 | 0.012 |

In this table, the regression coefficient of VIX index is 0.08, while the significance level is 1%. These results clearly show that when the VIX index rises 1 unit, the market crash risk measurement index (negative skewness index) will rise 0.08 units, strongly confirming the positive correlation between the VIX index and the risk of crash. That is to say, as market panic intensifies, the risk of market crashes increases dramatically, reflecting a rise in the VIX.

Market liquidity index regression coefficient was - 0.12 which was significant at 5% significance level. For every 1 unit increase in market liquidity, the stock market crash risk measurement index decreases 0.12 units, which shows that good market liquidity can decrease the risk of market crash. When market activity is active with sufficient liquidity, stock price fluctuations are relatively stable, market flexibility is more flexible, reducing crash risk.

The regression coefficient of GDP growth rate is - 0.06, which is significant in 5% significance level. For every 1 percentage point of growth of GDP, the stock market crash risk measure index decreases 0.06 units, indicating that macroeconomic growth has a significant effect on the risk of stock market crash. Stable economic growth provides good development environment for enterprises, enhances market confidence, helps to stabilize stock market, and reduces crash risk.

3.4. Robustness test

In order to ensure the reliability and stability of the research results, this study uses a variety of methods for robustness testing.

3.4.1 Substitution variable method

First, the substitution variable method is used to replace the original negative skewness index as the stock market crash risk measurement index calculated by the method proposed by Campbell and other scholars, and the regression analysis is re-performed. The results are shown in Table 4 below:

Table 4 Regression results of substitution variable method robustness test

| explanatory variables | Regression coefficient | Standard error | t value | P-value |
|------------------------|------------------------|----------------|---------|---------|
| VIX index | 0.15 | 0.03 | 5.00 | 0.000 |
| Market liquidity index | -0.18 | 0.06 | -3.00 | 0.003 |
| GDP growth rate | -0.08 | 0.04 | -2.00 | 0.046 |
| Constant term | -0.20 | 0.08 | -2.50 | 0.012 |

It can be clearly seen from the table that even if the stock market crash risk measurement indicator is replaced, the VIX index and the new crash risk measurement indicator still show a significant positive correlation, and the regression coefficient is still statistically significant. This shows that the positive correlation between the VIX index and the stock market crash risk does not depend on a specific risk measurement indicator and has strong robustness.

3.4.2 Changing the sample interval method

Secondly, the changing sample interval method is adopted to shorten the research period to January 1, 2005 to December 31, 2015, and conduct empirical analysis again. The specific results are shown in Table 5 below:

Table 5 Regression results of the robustness test of the changing sample interval method

| explanatory variables | Regression coefficient | Standard error | t | P |
|------------------------|------------------------|----------------|-------|-------|
| VIX index | 0.09 | 0.02 | 4.50 | 0.000 |
| Market liquidity index | -0.13 | 0.05 | -2.60 | 0.009 |
| GDP growth rate | -0.07 | 0.03 | -2.33 | 0.020 |
| Constant term | -0.16 | 0.06 | -2.67 | 0.008 |

The test results show that in the new sample interval, the positive correlation between the VIX index and the risk of stock market crash remains stable. This further shows that the research results are not greatly affected by the selection of the sample interval and have high reliability and stability. Through these two robustness test methods, the reliability of the results of this study is fully verified, the persuasiveness of the research conclusions is enhanced, and a solid and reliable basis is provided for subsequent related research and market practice applications.

4. Conclusion

This paper makes a deep research on the dynamic correlation between VIX index and stock market crash risk, and draw some conclusions by simulation analysis. For data selection, VIX and S&P 500 index data for January 1, 2000 through 31 December 2020 are included. According to descriptive statistics, the mean value of VIX index is 19.56; standard deviation is 9.87; the mean value of negative skewness is -0.25; stock return distribution is left skewed with risk of crash. According to correlation analysis, Pearson correlation coefficient between VIX index and negative skewness index was 0.68, with a significant positive correlation at 1% significance level. There is a significant negative correlation between market liquidity and GDP growth rate and negative skewness index. Multivariate linear regression model indicates that the regression coefficient of VIX index is 0.08 at 1% significance level, which indicates a significant increase in risk measurement index of 0.08 units for every 1 unit increase; market liquidity index regression coefficient -0.12; GDP growth rate regression coefficient -0.06; both are significant at 5% significance level. The robustness test verifies that VIX index is robust to market crash risk by replacing variables and changing sample interval.

References

- [1] Zhang, X. Zhang, W. & Wang, C. Tail risks in developed and emerging markets. Chinese Journal of Management Science, Vol. 32(2024) No. 4, p. 14-25.
- [2] Zhou, Y. & Bei, Z. Dynamic price discovery and asymmetric volatility spillover between China's treasury bond futures and spot markets. Acta Econometrica Sinica, Vol. 1(2021) No. 4, p. 814.

- [3] Ma, T. Zhang, X. & Li, Z. Option implicit information and price discovery: A study based on China's on-exchange options market. *Journal of Financial Research*, Vol. 523(2024) No. 1, p. 169-186.
- [4] Chen, Y. Liu, Y. & Zhu, R. Analysis of the predictive ability of VIX on stock market return volatility: A case study of Hong Kong stocks. *Journal of Chaohu University*, Vol. 23(2021) No. 3, p. 27-37.
- [5] He, J. & Zou, X. Economic uncertainty measurement based on Weibo big data and its impact on macroeconomics. *Journal of Chongqing University (Social Science Edition)*, Vol. 28(2022) No. 5, p. 61-72.
- [6] Wang, X. Huang, D. & Gao, M. Volatility index and price discovery: theoretical expansion based on the Chinese market. *Financial Research*, Vol. 530(2024) No. 8, p. 113-131.
- [7] Dong, W. Tan, X. Zhu, F. & Li, X. Will US monetary policy affect the risk-taking of lenders in other economies? - A study based on the global syndicated loan market. *Financial Research*, Vol. 513(2023) No. 3, p. 57-73.
- [8] Zhang, R. Ma, R. Guo, X. & Wu, L. Research on the impact mechanism and time-varying nature of financial uncertainty on China's systemic financial risk. *Economic Research*, Vol. 43(2022) No. 6, p. 88-106.