Adaptive Asset Allocation Under Abrupt Stock-Bond Correlation Reversals Using the Markowitz Framework

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Abstract. Markowitz Mean-Variance Model was proposed by Harry Markowitz. As a modern portfolio theory with important practical application value, the core content of Markowitz mean-variance model includes that investors can reduce non-systemic risks and help investors pursue higher returns by diversifying investment. This paper emphasizes how the theory can be applied flexibly under the actual extreme market. Firstly, this paper analyzes the correlation between assets through the example of the stock and bond market, find the reason for the traditional negative correlation between stocks and bonds. And then discusses the abnormal correlation between stocks and bonds triggered in extreme market periods. Next , essay focus on how investors judge the market environmental conditions, and then use the diversification of investment in Markowitz's theory to reduce risk. The discussion reveals that, the factors contributing to the weakening of the correlation between stocks and bonds include sudden changes in monetary policy, economic recession or rising inflation rates. Even worse, there is a possibility that the correlation could turn from negative to positive. When the correlation between assets changes, the return rate of traditional portfolio investors is greatly affected, and investors need to re-optimize the asset weight to reduce risk or increase the sharpe ratio. And At such times, the hedging and stable income-generating functions of traditional investment portfolios are weakened, and their effectiveness drops significantly. In order to continue to effectively avoid risk in extreme market environments, this article will discuss the importance of inflation-resistant assets, and suggest adding alternative assets such as gold and commodities to your asset allocation to hedge investment risks.

Keywords: Markowitz Mean-Variance Model; Stock-bond correlation; Anti- inflationary investment portfolio; Portfolio Diversification; Sharpe Ratio; Risk Management; Economic Recession.

1. Introduction

The Markowitz Mean-Variance Model proposed by Harry Markowitz in 1952 laid the foundation for modern portfolio theory, which is widely used in many financial fields such as asset allocation, hedge funds, and investment management. On the basis that the expected return of a portfolio is equal to the weighted average return of assets, Markowitz put forward the principle of diversified investment. Institutional investors can calculate and analyze the correlation between assets through covariance matrix, and expand diversification on the basis of combining low-correlation assets, so as to reduce investment risks and make modern investment decisions more scientific.

In the financial market, investors can make use of the stable negative correlation between assets to make this model play a high investment return value. Assets with common negative correlation, such as stocks and bonds, and major factors such as economic cycle and interest rate changes, make the stock and bond market have a seesaw effect. Investors can make a portfolio investment between the two based on this feature. For example, the 60/40 investment strategy emphasizes that investors can place 60% of the weight of assets in stocks to obtain long-term income, and the remaining 40% of the weight in bonds to provide stable returns and hedge functions when necessary. However, the correlation of the stock and bond markets is not fixed, Ma Huihong pointed out that, "From the perspective of the market pattern, the intensification of geopolitical competition and the continuous impact of inflation have impacted the supply chain and returned strategic capital, resulting in the traditional 60/40 portfolio being unable to cope with market fluctuations." If stocks and bonds double during the US interest rate hike from 2022 to 2023, the combination is difficult to return. (*China Securities Journal 2025)* "In the case of high inflation, financial crisis or policy upheaval, Investors need to re-weight their assets, and they can introduce gold as an important part of diversified asset

allocation in order to achieve the purpose of relying on diversification of assets to obtain ideal returns in Markowitz's theory." Gold has many advantages such as risk diversification, preservation and appreciation, hedge property and strong liquidity, and the combination of gold and other low-correlation assets also helps to build a stable and effective investment portfolio (*21st Century Business Herald 2025*).

Most studies have focused on the hypothesis of stable negative correlation between stocks and bonds, but few articles have focused on its sudden correlation change and countermeasures under extreme market environment. This paper is divided into three parts. The first part explains the mathematical formula and application principle of Markowitz Mean-Variance Model, and analyzes the reasons for the negative correlation of the stock and bond markets. The innovation of this paper is reflected in the second part, which will analyze how macroeconomic factors lead to changes in the correlation of stocks and bonds, and show the time-varying correlation scenario of stocks and bonds based on the global stock and bond market data from 2008-2023. The third part includes the construction of the investment model after the introduction of inflatable assets, and introduces the variance -minimization weighting model and the stock-debt recalculation when Sharp ratio is maximized. The final part puts forward the suggestion of investment rebalancing strategy.

2. Methodology

This paper aims to use Markowitz theory to calculate the correlation fluctuations of stock and bond markets combined with relevant years, and introduce a simple strategy on how to optimize asset allocation in the latter part. In order to achieve this goal, IImanen (2003), Henry Neville, The relevant literature of Teun Draaisma et al. analyzed the correlation factors affecting the stock and debt markets. By extracting the year-price data of S&P 500 and iShare 7-10 year Treasury bond ETF, Markowitz mean variance model can be used in calculating its correlation coefficient. Then, correlation analysis is carried out to complete the verification of the sudden change in the correlation of stocks and bonds, which was caused by the extreme market environment in specific years. Using Sharpe ratio formula, the asset weight when Sharpe ratio is maximized is calculated. In addition, under the real investment market environment in a specific year, the simulated investor makes the choice of traditional investment portfolio and anti-inflation investment portfolio respectively. During the period, the change trend of the net value of the two portfolios is also be calculated. Finally, the risk diversification and investment return of anti-inflation assets under extreme market environment are evaluated by comparing the net value curves.

3. Literature review

The development of modern investment theory and the principle of Markowitz Mean-Variance Model

Due to the lack of systematic investment methods and theoretical references, early investors often make investments based on experience, intuition and even market sentiment. Traditional investment methods, such as the value investment method proposed by Benjamin Graham based on the concern about the intrinsic value of enterprises, enable investors to pay attention to the profitability and development trend of subordinate companies behind a single asset. It can be shown that early investors relatively rely on personal preferences and subjective judgment, even through the analysis of individual stocks, but also ignore the correlation between the assets of the portfolio, so when the overall market appears a downward trend, investors can not avoid the risk of significant losses. It was not until Harry Markowitz put forward Modern Portfolio Theory in his paper Portfolio Selection that the development of modern investment theory reached a significant milestone. The most important aspect of Markowitz' model was his description of the impact on portfolio diversification by the number of securities within a portfolio and their covariance relationships (*Megginson, 1996, p. 325*).

The theory subsequently attracted wide attention in the financial investment community, and Markowitz won the Nobel Prize in Economics in 1990 for its initial construction.

In the Markowitz Mean-Variance Model, risk can be regarded as the volatility between different assets, which can be measured by standard deviation and variance. The formula is as follows:

$$\sigma\rho^{2} = \sum \mathrm{Wi}^{2}\sigma\mathrm{i}^{2} = \sum \sum \mathrm{Wi}\,\mathrm{Wj}\,\sigma\mathrm{ij}$$

Where, $\sigma\rho^{2}$representing the total risk of the portfolio, $\sigma\mathrm{i}^{2}$representing the variance of individual assets, $\sigma\mathrm{ij}$ is the covariance between asset i and asset j.

Correlation between assets is an important index in a portfolio. It measures the price trend relationship between two or more assets and is obtained by the covariance ($\mathrm{Cov}(\mathrm{X},\mathrm{Y})$) and standard deviation ($\sigma\mathrm{X}\sigma\mathrm{Y}$) between assets. It is usually displayed in the mathematical form of Pearson correlation coefficient, with the interval between -1 and 1:

$$\rho\mathrm{x},\mathrm{y} = \frac{\mathrm{Cov}(\mathrm{X},\mathrm{Y})}{\sigma\mathrm{x}\,\sigma\mathrm{y}}$$

According to Pearson correlation coefficient, the correlation between assets can be divided into three states. When the correlation coefficient (ρ) is between 0 and 1, assets are positively correlated, which is reflected in the rise and fall of asset prices. When the correlation coefficient(ρ) is between -1 and 0, the assets are negatively correlated, and the price of one side goes up while the price of the other side goes down. A correlation coefficient (ρ) of 0 means that there is no obvious correlation between the price changes of assets. The terms 'diversification' and 'Diversification Effect' refer to the relationship between correlations and portfolio risk. *(Myles E. Mangram, SMC University, Switzerland)*, investors can reduce the impact of price fluctuations in a single stock market by combining assets with low or negative correlation.

Decisive factors of correlation in the stock and bond market

Under ceteris paribus conditions, favorable macroeconomic conditions lead to increased corporate profitability, which drives demand for stocks and stock prices. While when the economy enters a recession, investors seek to buy bonds to avoid risk, driving bond demand and prices higher. When the risk-off effect appears, it will lead to a negative correlation between stocks and bonds. When there are risk events or increased uncertainties in the market, negative stock-bond return correlations to flight-to-safety (FTS) episodes, during which increased stock market uncertainty induces investors to flee stocks in favor of bonds.(*Connolly et al. (2005), Baur and Lucey (2009)*) At the same time, investors selling stocks and buying bonds and other safe assets will cause a seesaw effect in the stock and bond markets, resulting in falling stock prices and rising bond prices, thus strengthening the negative correlation between the stock and bond markets. In particular, the collapse of financial stock markets made investors more risk-averse as they moved money out of stocks and into the bond market. (*Nebojsa Dimic, Jarno Kiviaho, Vanja Piljak, Janne Aijo, 2015*). In IImanen and Antti's view, such a negative correlation is attractive because, while bond risk premiums are very low, government valuations are high, because the negative correlation makes government bonds an excellent hedge against major systemic risks, such as recession, deflation, stock market weakness, and other financial market crises.

According to Campbell & Ammer, they believe that the negative correlation between stocks and bonds can be explained by the discounted cash flow model. The value of stocks and bonds is mainly determined by the interest rate (Discount rate). For example, the interest rate hike of the Federal Reserve will increase the real interest rate expected by the market, and the present value of bonds will decrease due to the rise in discount rate. On the other hand, the value of stocks is more complicated, and it is necessary to consider whether the interest rate hike is due to overheating of the economy. If the rise in the discount rate outweighs the rise in cash flow, the price of the stock may fall. Boyd et al. (2005) and Andersen et al. (2007) also studied the effects of macroeconomics on stock and bond markets during expansions and recessions. They argue that the cash flow effect may dominate during deflation, while the discount rate effect may be more important during expansions, resulting in a

positive correlation between stock and bond returns during expansions and a low or even negative correlation during recessions. Ilmanen (2003) makes a similar argument. Further, Burkhardt and Hasseltoft (2012) relate the correlation between stocks and bonds and inflation (pro-cyclical or counter-cyclical). When inflation is procyclical, equity and bond risk premia are negatively correlated as nominal bonds act as a hedge, inducing a negative stock-bond correlation.

Macroeconomic factors that lead to changes in the correlation between stocks and bonds

The returns from portfolio diversification depend heavily on correlations between assets, But The relation between the two main asset classes has not been particularly stable. The correlation has tended to be positive but has occasionally dipped below zero for extended periods. 1929-1932, 1956-1965 and 1988-2001 are three periods of negative correlation between stocks and bonds. (*IImanen & Antti 2003*). Understanding the time-varying nature of stock-bond correlations is of critical importance for asset allocation and risk management, as they are the two most important asset classes (*Connolly et al., 2005, D'Addona & Kind, 2006, Kim et al., 2005). 2006*) For example, the average US stock - bond return correlation has varied from +0.2 during 1968-1978 to +0.4 during 1980-1998 before turning −0.2 during 2001 − 2007 (*Baele et al.2010*).Shen and Weisberger's report points out that even though the negative correlation between US stocks and bonds has continued since the early 2000s, it is necessary to be alert. Changes in macroeconomic conditions related to monetary and fiscal policy can be the driving factors that push the correlation from negative to positive.

Andersson et al., 2008, Campbell and Ammer, 1993, Cappiello et al., 2006 In this series of literatures, various factors driving stock-bond correlation were studied. Given the mixed evidence in the literature on the role of macroeconomic factors in driving the correlation between stocks and bonds, the debate on this issue remains open (*Nebojsa Dimic,Jarno, Kiviaho,Vanja, Piljak,Janne, Aijo*). It is worth noting that some of this literature documents the importance of several important macroeconomic elements, in particular inflation, the business cycle environment, and the stance of monetary policy in explaining the correlation between stocks and bonds. Andersson et al. (2008), based on the data of the stock and bond markets in the UK, Germany and the US, found that economic growth expectation was not the relevant factor driving the change of the correlation between stocks and bonds, while inflation expectation was an important determinant of the correlation between stocks and bonds. According to their research results, when inflation expectation was high, the correlation between stocks and bonds would increase. Stock and bond prices move in the same direction.

If there is an unexpected surge in inflation, the expected inflation embedded in the yield increases and the bond price usually falls(*Henry Neville, Teun Draaisma*). During periods of high inflation, central banks raise interest rates to curb persistent inflation, and rising interest rates cause yields on issued bonds to rise. According to the formula of market interest rate (Y) and bond price (P), we can see:

$$P = \sum_{t=0}^{n} \frac{C}{(1+Y)\hat{}t} + \frac{F}{(1+Y)\hat{}n}$$

When market interest rates rise, bond prices (P) will fall because of the depreciation caused by the rise in interest rates. The same is true of stock valuations, which have fallen. But the impact of a high-inflation environment on stocks is more complex and dynamic.

Baele, Lieven, and Frederiek Van Holle(*2017*) also find stock-bond correlations to be lower and often negative in low inflation regimes, while they are positive and large in intermediate and high inflation regimes. In their empirical study, by estimating the integrated version and the subdivided regional version of the market model, it is found that the global macroeconomic mechanism, not only the local macroeconomic mechanism, drives the change of the correlation between stocks and bonds. While Baele et al. (2017) associate low inflation with negative stock-bond correlations, Belle et al. (2017) argued that the negative correlation between low inflation and stocks and bonds is related. However, contradictory evidence emerged from studies on emerging markets. For instance, Garcia and Silva (2020) observed a persistent positive correlation during periods of low inflation in Brazil. They attributed this to the distortion of bond yields caused by central bank intervention. This contrast

highlights that the macroeconomic framework and policy environment may override the impact of inflation on the correlation.

Henry Neville, Teun Draaisma et al. (*May 25, 2021*), through data collection and empirical evidence, summarized their main findings for the US and ranked the various passive and dynamic strategies by the annualized average real returns during the eight inflationary regimes, finding that bond prices have fallen has been accompanied by poor stock market performance, with the rise in stock-bond correlations compounding the challenges faced by investors in traditional 60/40 bond portfolios.

Time-varying correlation scenario of stocks and bonds based on 2008-2023 global stock and bond market data

The correlation between stocks and bonds is time-varying and not completely stable. In order to show the fluctuation of the correlation between stocks and bonds since 2008, the following table and line chart report are developed by referring to the monthly yield of S&P 500 and iShare 7-10 year Treasury bond ETF and using Markowitz Mean-Variance Model to calculate the correlation:

Note: Data from Yahoo Finance, excerpts of adjusted closing prices of S&P 500 and IEF ETF securities, covering January 2008 through January 2023. The yield calculation is based on the adjusted closing price, with a rolling window of twelve months. Using Excel as a calculation tool, the formula for calculating monthly return rate is as follows:

$$Rt = \frac{Pt - P(t-1)}{P(t-1)}$$

Where, Rt is the t month return, Pt is the asset price at the end of t, and P(t-1) is the asset price at the end of t month.

The covariance formula is:

$$Cov(X,Y) = \frac{1}{n-1}\sum_{i=1}^{n}(Xi - \bar{x})(Yi - \bar{y})$$

The standard deviation formula is:

$$\sigma x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(Xi - \bar{x})^2}$$

The formula for calculating the correlation coefficient of the two assets is as follows:

$$\rho x,y = \frac{Cov(X,Y)}{\sigma x\, \sigma y}$$

Table 1. The correlation varies across different years

Year	Correlation coefficient of return
2008	-0.84
2009	0.02
2010	-0.73
2011	-0.66
2012	-0.56
2013	-0.4
2014	-0.6
2015	-0.75
2016	-0.65
2017	-0.5
2018	-0.8
2019	-0.3
2020	-0.51
2021	0.204
2022	0.65
2023	0.68

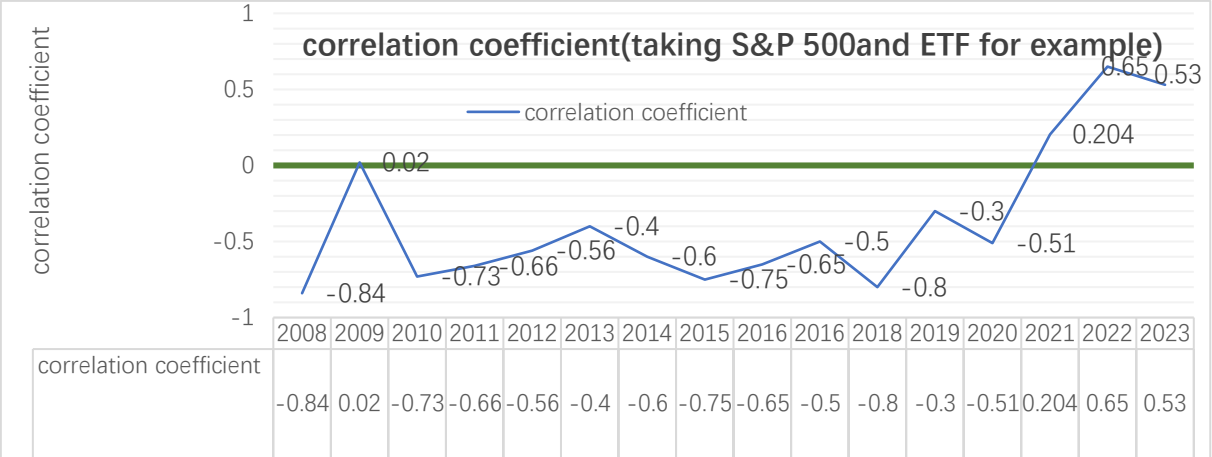

Figure 1. The correlation between stocks and bonds changes over years

Through the above table and chart, we can intuitively feel the significant changes in the correlation between stocks and bonds in the two periods from 2008 to 2023. In 2008, the world experienced the great financial crisis, investors were hit by the extreme market and sold stocks one after another, the S&P 500 fell by nearly 40% for the whole year, and the US medium-term Treasury bond ETF was bought in large numbers because it had the traditional hedge advantage of bonds, and the price rose, so the correlation between stocks and bonds increased significantly to -0.84, which was near the bottom. The characteristics of investors using the negative correlation between stocks and bonds to avoid risks are played. In 2009, the Federal Reserve adopted quantitative easing policy, the stock market and investor confidence began to recover, the bond market fell back, investors reallocated investment assets, and the S&P 500 gained about a quarter of the year. The correlation between stocks and bonds has also recovered from an extreme state to near zero or slightly positive.

From 2010 to 2020, the correlation of the stock and bond markets remains around -0.5, which is always negative. In 2022, a special "double kill" phenomenon occurred, Because the Board of Governors of the Federal Reserve System voted unanimously to approve a 1/2 percentage point increase in the primary credit rate to 4.5 percent, effective December 15, 2022. (Federal Reserve,2022) The Fed's aggressive rate hike has had a series of effects on the stock and bond markets, The stock market and the bond market fell at the same time, of which the price of long-term bonds was the most affected, the S&P 500 index fell nearly 20% for the whole year, until the inflation decline in 2023 prompted the rate hike policy to slow down, and the stock and bond market had a rebound trend.

However, The above data analysis may have limitations and biases. The data only focuses on the US market (selecting S&P 500 and Treasury ETF), which may overlook the possibility of differences existing in emerging economies. Meanwhile, during the calculation process, using monthly data as a reference conceals the possible sensitive fluctuations within each month, reducing the accuracy of the model in considering sudden market situations.It is suggested that other researchers extract sufficient data for improvement and supplementation.

According to Markowitz's theory, when the correlation between stocks and bonds turns from negative to positive, the efficiency of traditional diversification is reduced, and investors need to find other related assets to ensure the diversification of risks and obtain higher returns during the period of market changes.

variance minimization weight model

According to Markowitz's mean-variance model, the risk of an asset can be quantified by the variance of the asset's return, which is reflected in the volatility of the return. Portfolio risk not only depends on the volatility of assets (σ_s, σ_b), but also the correlation between assets is an important indicator. Therefore, when the market conditions change, it is necessary to recalculate the covariance matrix and optimize the weights. The previous analysis shows that under extreme market conditions (such as high inflation in 2022), traditional diversification strategies fail. In order to meet this

challenge, the asset weights of stocks and bonds are dynamically calculated based on the Minimized variance weight model to find the maximum reduction in portfolio volatility.

According to the Markowitz mean-variance model, and based on two assets (stocks and bonds), the stock weight is w_s, and the bond weight is $w_b = 1 - w_s$,the global minimum variance portfolio (GMVP) stock weight solution is

$$w_s = \frac{\sigma^2 - \rho\sigma_s\sigma_b}{\sigma_s^2 + \sigma_b^2 - 2\rho\sigma_s\sigma_b}$$

σ_s, σ_b: Volatility of stocks and bonds.

ρ: Correlation coefficient between stocks and bonds.

* Constraint: $0 \leq w_s \leq 1$

The global minimum variance portfolio explicitly defines risk as variance and is a simplified version of Markowitz's theory that simply optimizes the portfolio variance to represent the least risky asset allocation of all portfolios, at the leftmost end of the efficient frontier. Variance minimization weight calculation is aim to minimize risk by identifying the portfolio with the lowest volatility, thereby protecting investors' principal in an uncertain market

Based on the above formula, the S&P 500 and US bond ETF are selected as examples, and the data from 2008-2011, 2020-2023 are used to calculate the weight table of the minimum risk of stock and bond portfolios in each year:

Table 2. Annual return of variance minimization investment portfolio

Year	Stock weight	Bond weight	Portfolio return
2008	26%	74%	-3.28%
2009	13%	87%	3.46%
2010	25%	75%	10.04%
2011	23%	77%	13.3%
2020	11%	89%	6.62%
2021	8%	92%	-1.68%
2022	0%	100%	-10.22%
2023	0%	100%	-2.2%

The following is the return rate of the traditional stock bond portfolio (60% stocks +40% bonds) in the two sets of time periods from 2008 to 2010,2020 to 2023:

Table 3. Annual return of traditional investment portfolio

Year	Stock weight	Bond weight	Portfolio return
2008	60%	40%	-20.14%
2009	60%	40%	17.8%
2010	60%	40%	14.57%
2011	60%	40%	7.89%
2020	Stock weight	Bond weight	13.1%
2021	60%	40%	15.5%
2022	60%	40%	-14.95%
2023	60%	40%	13.64%

Through the comparison of portfolio return rate, we can know that the calculation of variance minimizationweight helps investors reduce the total risk of the portfolio to the maximum effect, so that its return tends to be more stable.

Bonds are generally less volatile than stocks, so increasing the weighting of bonds can reduce the overall risk of an asset. During the 2008 financial crisis, for example, the stock market plunged, with the S&P 500 falling nearly 40% for the year, while U.S. bond ETFs rose in price, yielding an annual return of about 9.6%. According to the variance minimization weighting model, to hedge the risk of a stock market crash, bonds should have a weighting of more than 70%, with a portfolio return of -

3.28%. Compared with the -20.14% return of a traditional portfolio with a 40% bond weighting, the former significantly reduces portfolio volatility and avoids severe principal losses.

In 2022, when the Federal Reserve raised interest rates and there was a period of high inflation, the S&P 500 index fell 18.1% and ETF bonds fell 10.2%. The calculation of variance minimization weight shows that if investors make a portfolio choice without stock weighting and buy bonds in full position, the portfolio loss is smaller than the traditional portfolio.

Limitations of the variance-minimization weighting model

Although the model of variance-minimization weight suggests that investors should tilt their investment weight toward bonds in the period of sharp market fluctuations or high inflation, it only focuses on volatility and easily ignores investors' goal of pursuing returns. For example, in the two sets of data in 2021,2023, the weighting scheme provided by the variance minimization weight model finally presents negative returns. Unable to meet investor demand. However, in 2022, when the correlation between assets changes abruptly and the rate of return goes down simultaneously, this model does not provide effective protection.

Continue to introduce the concept of Sharpe ratio, developed by William F. Sharpe, which aims to show the return of an asset portfolio relative to risk.

The formula is:

$$S = \frac{R_p - R_f}{\sigma_p}$$

S is the Sharpe ratio, R_p is the expected rate of return of the portfolio, R_f is the risk-free rate of return (taking the US historical Treasury bond yield of 3% as an example), σ_p is the standard deviation of the portfolio return.

The numerator of the Sharpe ratio can be seen as the excess return of the portfolio relative to the risk-free asset, and the denominator can be seen as the volatility (risk) of the portfolio, so the Sharpe ratio is used to measure the excess return received per unit of risk. The higher the Sharpe ratio obtained by calculation and statistics, the higher the return brought by unit risk, which can be used to judge whether the investment is effective. Usually, if the Sharpe ratio is greater than 1, it is judged that the portfolio has a higher excess return; if the Sharpe ratio is less than 0, the return brought by the portfolio is lower than the risk-free interest rate, indicating that the investment is invalid, and risk-free investment can be selected instead, such as stable return. A short-term government bond that guarantees the investor's principal.

Through the above formula, the S&P 500 and the US bond ETF are selected as examples, and the data from 2008-2011, 2020-2023 are still used to calculate the Sharpe ratio table of the least risky stock and bond portfolios in each year:

Table 4. Annual Sharpe ratio of variance-minimization investment portfolio

Year	Stock weight	Bond weight	Sharpe ratio
2008	26%	74%	-1.28
2009	13%	87%	0.26
2010	25%	75%	6.60
2011	23%	77%	10.03
2020	11%	89%	3.18
2021	8%	92%	-3.23
2022	0%	100%	-4.24
2023	0%	100%	-1.95

Through the calculation of Sharpe's scale, it can be found that the weight configuration recommended by the variance-minimization weight model is negative for the years indicated above. This indicates that the return of these investments is lower than the risk-free rate of return, the excess return cannot compensate for the risk, and investors will suffer more losses after taking risks, so these asset portfolios perform poorly in the market environment.

Sharpe ratio maximization weight calculation

In order to continue to optimize asset allocation and maximize return per unit risk, investors are required to readjust their asset weights, using the following formula:

$$\underset{w}{MAX}\ SR = \frac{E(R_p) - R_f}{\sigma p}$$

For :

$$E(R_p) = \mathrm{w_s} * \mathrm{R_s} + (1 - \mathrm{w_s}) * \mathrm{R_b}$$

$$\sigma p:\ \sqrt{w^2 \sigma_s^2 + (1 - \mathrm{w_s}) \sigma_b^2 + 2w\,(1 - \mathrm{w_s}) \rho \sigma_s \sigma_b}$$

R_f: Risk-free interest rate

In this case,

$$w_b = \frac{E(R_b)\sigma^2 - E(R)Cov(RR)}{E(R_b)\sigma^2 + E(R_s)\sigma^2 - [E(R_b) + E(R_s)]Cov(R_b R_s)}$$

$$\mathrm{w_s} = 1 - \mathrm{w_b}$$

The calculated data are as follows:

Table 5. Annual Sharpe ratio of Sharpe ratio maximization investment portfolio

Year	Optimal stock weight	Optimal bond weight	Sharpe ratio
2008	0%	100%	2.22
2009	100%	0%	4.60
2010	29%	71%	6.94
2011	0%	100%	13.30
2020	18%	81%	3.53
2021	100%	0%	6.88
2022	0%	0%	0
2023	89%	11%	6.43

* Note: The expected rate of return of all assets in 2022 is risk-free interest rate, so choose not to hold any risk assets between stocks and bonds.

In 2008 and 2011, negative returns on the S&P 500 reduced the expected return on assets, so all-bonds became a viable option with the highest sharpe ratio. On the contrary, in 2009 and 2021, the negative return rate of bond ETFs will reduce the expected return rate of assets, and in 2021, there is a positive correlation between stocks and bonds, which weakens the diversification effect, holding bonds only increases portfolio volatility without compensation for returns, so stocks can maximize the return brought by unit risk.

However, the asset weight under Sharpe's maximum ratio often leads to the concentration of funds on a single asset or some assets, resulting in the neglect of the risk reduction effect brought by portfolio diversity, which makes investors face risk uncertainty when making concentrated investment choices only on the historical return rate and correlation data of assets under the unknown market environment in reality.

The stock market rally that occurred in 2009 (the S&P 500 returned 30.03%) was based on the Federal Reserve's quantitative easing policy, and the results of Sharp's scaling maximization model suggest holding a full position in stocks to capture the gains. Although variance minimization and Sharp ratio model can inspire investors to reduce risks or improve weighting schemes to maximize returns to a certain extent, they rely on historical data to predict future returns and ignore tail risks brought by structural changes in the market, so it is difficult to allocate according to model weights in practice. In the 2022 stock and bond sell-off, the return of both types of assets is lower than the established 3% risk-free rate, and the Sharp ratio model recommends the liquidation of risk assets. This phenomenon demonstrates the limitations and vulnerabilities of traditional asset allocation in

extreme market conditions, and the need to introduce anti-inflation assets (e.g., gold, commodities) to construct diversification to compensate for the inadequacies of the strategy.

Suggestions on rebalancing strategy

Different risk preferences of investors may drive investors to make different asset weight choices. The variance minimization model may be more useful for investors who are risk averse or unwilling to bear excessive losses, while investors who are risk averse or focus on high returns will pay more attention to the reference significance brought by Sharpe ratio.

Through the comparison of the above data, the objective of the two weight optimization methods cannot fully take into account the risks and benefits in the extreme market. This suggests that investors need to continue to introduce assets that have a low correlation with stocks and bonds and can provide positive returns.

Henry Neville and Teun Draaisma calculated the average annual real return data of commodities during periods of inflation and found that commodities tend to perform best during periods of high inflation, such as energy (oil, natural gas), industrial metals (copper, aluminum), and precious metals (gold, silver). Overall, in the eight periods of high inflation in the US they analysed, commodities consistently produced positive real returns, averaging a real return of +14 per cent a year. This shows that commodities can provide investors with some investment protection in times of inflation. In the face of a double-kill stock and bond market similar to 2022, the correlation between stocks and bonds suddenly becomes positive, and investors in the traditional 60/40 stock bond portfolio can buy commodities to hedge certain market risks and guarantee returns.

Calculate risk diversification and increase the degree of return by introducing hard assets (gold as an example)

As an inflation-proof asset, gold is a good basis for diversification portfolios, both from the standpoint of the risk-averse investor and from the aspect of the investor prepared to take a higher. Take 2022 as an example, during the period of high national inflation, the S&P 500 stock market fell seriously, while in contrast, gold fell only 1%, so here we collect the S&P 500 price (2021-2023), The stock ETF price (2021-2023) and the gold price (2021-2023) construct the traditional 60/40 bond portfolio and the anti-inflation asset portfolio after the introduction of gold, and after reallocating the weight, the investment proportion is 50% stocks, 30%ETF securities, and 20% gold. Finally, by calculating the net value of the two portfolios at the end of 2023, the risk diversification and return efficiency of the two portfolios are compared.

The portfolio selects the S&P 500 Index (stocks), IEF EFF (bonds) and London gold as three asset representatives. The gold data is derived from the London Bullion Market (LBMA) website and extracts the London Gold fixed price (USD/ ounce) at the end of the month published by the LBMA from January 2021 to November 2023. Use Excel as a calculation tool to calculate monthly returns. The initial net value of the asset portfolio for each time period is set to 1, using the net value formula:

$$\textbf{\textit{Net Value}}\ t = \textbf{\textit{Net Value}}(t-1)*(1+Rt).$$

This calculation is used to compare the performance of the Anti-Inflation Asset Portfolio and the Traditional 60/40 Stock-bond Portfolio.

Table 6. Net value changes of the two asset portfolios from 2021 to 2023

Year	The accumulated net value of the anti-inflation portfolio	Traditional portfolio cumulative net value
2021/1	1	1
2021/2	1.01	1.02
2021/3	1.05	1.05
2021/4	1.07	1.06
2021/5	1.07	1.08
2021/6	1.09	1.12
.....	.....	
2023/8	1.04	1
2023/9	1.02	0.97

2023/10	1.09	1.05
2023/11	1.13	1.09

Based on the time-net value data of the two investment portfolios in Exhibit 3, the following chart is drawn to reflect the growth, drawdown and overall returns of the two investment portfolios:

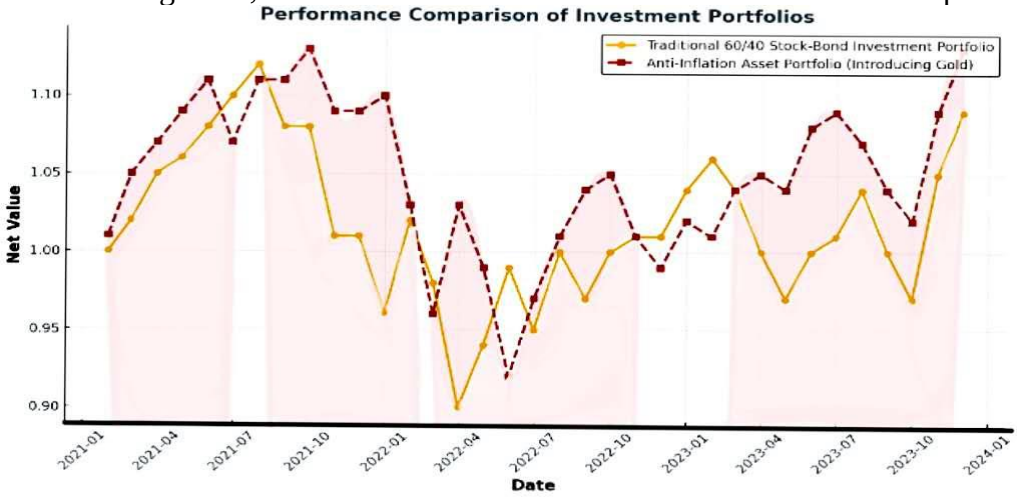

Figure 2. Net value tendency of the two asset portfolios from 2021 to 2023

Table 7. Net value of the two asset portfolios for specific years

Year	The accumulated net value of the anti-inflation portfolio	Traditional portfolio cumulative net value
2022/8	0.90	0.92
2022/9	0.94	0.97
2023/11	1.09	1.13

During the August 2022 sell-off, the net value of the traditional 60/40 portfolio fell to 0.90, while the net value of the 20% gold anti-inflation portfolio remained at 0.92, which shows that gold has a partial hedging role. At the same time, thanks to the increase of the London gold price by more than $110 / oz from August to November 2023, the net value of the anti-inflation portfolio with gold (1.13) still exceeds that of the traditional 60/40 portfolio (1.09) by November 2023, which supports the conclusion of Garcia & Silva (2020). It reflects the view that gold increases the resilience of asset allocation in times of high inflation.

From the above chart, we find that compared with the traditional 60/40 bond portfolio after the introduction of gold, the anti-inflation form asset portfolio (50% stocks, 30%ETF securities, 20% gold) has a higher net value for most of the time and a better ending net value and annualized return performance in most of the periods when the correlation of the bond market is unstable from 2021 to 2023. This reflects that gold partially offset the losses in stocks and bonds, because the price of gold is positively correlated with inflation in the long run, and can effectively hedge against inflation. Therefore, when economic crisis and inflation occur, gold's value preservation makes gold an important safe-haven asset. Although gold is a kind of anti-inflation assets, Barr and McDermott (2010) cautioned that during systemic liquidity crises such as the 2008 financial crisis, gold may not fully demonstrate its ability as a safe-haven asset, because in such large-scale financial crises, all assets will face selling pressure. Moreover, the storage cost of gold and its price fluctuations (for example, a 20% price drop in 2013) indicate that investors must weigh the relationship between their returns and actual restrictions before making corresponding investment decisions.

But even so, In another study, Tijana found that the difference between the bond index EG05 and gold correlation is only 0.08, while between the stock index EuroStoxx50 and gold correlation it is slightly negative, at -0.09. There is little correlation between gold and other financial assets, which shows the feasibility of including gold in a portfolio. When the stock market or bond market falls, the

price of gold can remain stable or rise. If investors include gold as part of the portfolio, it can effectively curb the volatility (risk) of the overall portfolio.

4. Conclusion

After analyzing the correlation of stock and bond markets in specific years, we find that economic crisis, policy turbulence and financial market uncertainty emphasize the importance of risk management and portfolio diversification in Markowitz's theory. In such extreme market conditions, the correlation between stocks and bonds becomes stronger. After comparing the net value curves of inflation-resistant and non-inflation-resistant portfolios, the results show that during the period of high inflation in 2022, the correlation between stocks and bonds changes from negative to positive, and the anti-risk effect of traditional portfolios decreases significantly, while the inflation-resistant portfolios with gold are more stable. This finding is consistent with the conclusions of historical literature (e.g. Baur & Lucy, 2010, Tijana Showoja 2019) regarding the important anti-inflation properties of gold in portfolios, which justifies the hypothesis of this paper. The paper suggests that during stable periods, the traditional 60/40 equity-bond allocation can be sustained. If the correlation between stocks and bonds exceeds +0.3 for three consecutive months (indicating the existence of systematic risks), it suggests that asset allocation needs to be dynamically adjusted, and the allocation of gold can be increased to 30%, while the allocation of stocks can be shrinked, such as reducing it to 40%. Furthermore, in case of severe inflation (Consumer Price Index (CPI) > 10%), investors need to formulate contingency plans. For instance, 15% of their funds can be allocated to energy commodities (such as oil exchange-traded funds (ETFs)) to hedge against stagflation risks.

However, the model construction of the empirical part of this paper still has the limitation of oversimplification, and still lacks the consideration of complete realistic factors. For example, in this paper, only some stocks and bonds are selected for data connection in the stock and bond market. In the introduction of anti-inflation assets, only gold is selected as a calculation demonstration, and other assets, such as real estate, silver and crude oil, are not introduced for complex demonstration. In terms of time span, this article uses monthly data for overall calculation and does not accurately capture the extreme volatility of the market in the short term. However, the current content of this paper has shown the dynamic changes in the correlation of stocks and bonds, and suggests that investors should pay attention to inflation expectations and policy signals, and increase anti-inflation assets such as gold in time to enhance the resilience of portfolios when the stock and bond markets enter a period of rising correlation. Future research should pay more attention to the weight of traditional assets and anti-inflation assets in the portfolio, and find out how to maximize the risk diversification benefits of Markowitz's theory under the circumstances of The Times.

Appendix table 8. Appendix table 8: S&p 500 and Bond prices, S&p and bond monthly yield

Date	S&p 500 price	Bond prices	S&p monthly yield	Monthly bond yield
2008/1/31	1378.55	60.24	-3.48%	0.93%
2008/2/29	1330.63	60.8	-8.11%	1.33%
2008/3/31	1222.7	61.61	11.69%	-2.45%
2008/4/30	1365.59	60.1	5.48%	-1.75%
2008/5/31	1440.36	59.05	-11.13%	1.12%
2008/6/30	1280	59.71	-0.99%	0.74%
2008/7/31	1267.38	60.15	1.22%	1.51%
2008/8/31	1282.83	61.06	-9.08%	-0.13%
2008/9/30	1166.36	60.98	-16.94%	-0.72%
2008/10/31	968.75	60.54	-7.48%	7.57%
2008/11/30	896.24	65.12	0.78%	4.81%
2008/12/31	903.25	68.25	-8.57%	-3.21%
2009/1/31	825.88	66.06	-10.99%	-1.01%
2009/2/28	735.09	65.39	8.54%	3.32%
2009/3/31	797.87	67.56	9.39%	-2.75%

2009/4/30	872.81	65.7	5.31%	-2.10%
2009/5/31	919.14	64.32	0.02%	-0.54%
2009/6/30	919.32	63.97	7.41%	0.84%
2009/7/31	987.48	64.51	3.36%	0.74%
2009/8/31	1020.62	64.99	3.57%	1.14%
2009/9/30	1057.08	65.73	-1.98%	-0.12%
2009/10/31	1036.19	65.65	5.74%	1.43%
2009/11/30	1095.63	66.59	1.78%	-4.58%
2009/12/31	1115.1	63.54	-3.70%	3.43%
2010/1/31	1073.87	65.72	2.85%	0.00%
2010/2/28	1104.49	65.72	5.88%	-1.00%
2010/3/31	1169.43	65.06	1.48%	1.64%
2010/4/30	1186.69	66.13	-8.20%	2.92%
2010/5/31	1089.41	68.06	-5.39%	3.07%
2010/6/30	1030.71	70.15	6.88%	0.93%
2010/7/31	1101.6	70.8	-4.74%	3.35%
2010/8/31	1049.33	73.17	8.76%	0.01%
2010/9/30	1141.2	73.18	3.69%	0.01%
2010/10/31	1183.26	73.19	-0.23%	-0.90%
2010/11/30	1180.55	72.53	6.53%	-3.70%
2010/12/31	1257.64	69.85	2.26%	0.49%
2011/1/31	1286.12	70.19	3.20%	-0.46%
2011/2/28	1327.22	69.87	-0.10%	-0.14%
2011/3/31	1325.83	69.77	2.85%	1.81%
2011/4/30	1363.61	71.03	-1.35%	2.52%
2011/5/31	1345.2	72.82	-1.83%	-0.49%
2011/6/30	1320.64	72.46	-2.15%	3.17%
2011/7/31	1292.28	74.76	-5.68%	4.65%
2011/8/31	1218.89	78.24	-7.18%	2.25%
2011/9/30	1131.42	80	10.77%	-1.26%
2011/10/31	1253.3	78.99	-0.51%	0.58%
2011/11/30	1246.96	79.45	0.85%	1.81%
2011/12/31	1257.6	80.89	4.36%	1.24%
2012/1/31	1312.41	81.89		
2020/1/31	3225.52	102.31	-8.41%	2.82%
2020/2/29	2954.22	105.2	-12.51%	3.75%
2020/3/31	2584.59	109.14	12.68%	0.27%
2020/4/30	2912.43	109.44	4.53%	0.34%
2020/5/31	3044.31	109.81	1.84%	0.05%
2020/6/30	3100.29	109.86	5.51%	0.86%
2020/7/31	3271.12	110.81	7.01%	-0.96%
2020/8/31	3500.31	109.75	-3.92%	0.34%
2020/9/30	3363	110.12	-2.77%	-1.38%
2020/10/31	3269.96	108.6	10.75%	0.34%
2020/11/30	3621.63	108.97	3.71%	-0.30%
2020/12/31	3756.07	108.64	-1.11%	-0.97%
2021/1/31	3714.24	107.59	2.61%	-2.43%
2021/2/28	3811.15	104.98	4.24%	-2.38%
2021/3/31	3972.89	102.48	5.24%	1.01%
2021/4/30	4181.17	103.51	0.55%	0.42%
2021/5/31	4204.11	103.94	2.22%	1.02%
2021/6/30	4297.5	105	2.27%	2.00%
2021/7/31	4395.26	107.1	2.90%	-0.39%
2021/8/31	4522.68	106.68	-4.76%	-1.60%
2021/9/30	4307.54	104.97	6.91%	-0.45%
2021/10/31	4605.38	104.5	-0.83%	1.09%
2021/11/30	4567	105.64	4.36%	-0.62%
2021/12/31	4766.18	104.99	-5.26%	-1.95%
2022/1/31	4515.55	102.94	-3.14%	-0.40%

2022/2/28	4373.94	102.53	3.58%	-4.06%
2022/3/31	4530.41	98.37	-8.80%	-4.25%
2022/4/30	4131.93	94.19	0.01%	0.61%
2022/5/31	4132.15	94.76	-8.39%	-0.87%
2022/6/30	3785.38	93.94	9.11%	2.94%
2022/7/31	4130.29	96.7	-4.24%	-3.88%
2022/8/31	3955	92.95	-9.34%	-4.73%
2022/9/30	3585.62	88.55	7.99%	-1.48%
2022/10/31	3871.98	87.24	5.38%	3.60%
2022/11/30	4080.11	90.38	-5.90%	-1.67%
2022/12/31	3839.5	88.87	6.18%	3.99%
2023/1/31	4076.6	92.42	-2.61%	-3.47%
2023/2/28	3970.15	89.21	3.51%	3.72%
2023/3/31	4109.31	92.53	1.46%	0.82%
2023/4/30	4169.48	93.29	0.25%	-1.48%
2023/5/31	4179.83	91.91	6.47%	-1.25%
2023/6/30	4450.38	90.76	3.11%	-0.65%
2023/7/31	4588.96	90.17	-1.77%	-0.74%
2023/8/31	4507.66	89.5	-4.87%	-3.14%
2023/9/30	4288.05	86.69	-3.46%	-1.94%
2023/10/31	4139.8	85.01	10.34%	4.54%
2023/11/30	4567.8	88.87	4.42%	3.48%
2023/12/31	4769.83	91.96	1.59%	0.63%
2024/1/31	4845.65	92.54	5.17%	-2.32%
2024/2/31	5096.27	90.39		

Appendix table 9. S&p 500 annual return and Bond ETF annual yield

Year	S&p 500 annual return	Bond ETF annual yield
2008	-40.09%	9.66%
2009	30.03%	-0.51%
2010	19.76%	6.80%
2011	2.04%	16.67%
2020	18.40%	5.16%
2021	28.71%	-4.32%
2022	-18.11%	-10.22%
2023	24.20%	-2.20%

Appendix table 10. Net value for two investment portfolios from 2021 to 2023

year	Gold price	Monthly gold yield	Traditional Stock and Bond Portfolio Monthly Return (60/40)	Anti-inflation portfolio (50% stocks +30%ETF+20% gold)	Traditional portfolio cumulative net value	The accumulated net value of the anti-inflation portfolio
2021/1	1850.3	-6.57%	0.60%	-0.53%	1	1
2021/2	1728.8	-0.76%	1.59%	1.25%	1.02	1.01
2021/3	1715.6	3.04%	3.55%	3.53%	1.05	1.05
2021/4	1767.7	7.78%	0.50%	1.96%	1.06	1.07
2021/5	1905.3	-7.02%	1.74%	0.01%	1.08	1.07
2021/6	1771.6	2.57%	2.16%	2.25%	1.10	1.09
2021/7	1817.2	0.05%	1.58%	1.34%	1.12	1.11
2021/8	1818.1	-3.36%	-3.50%	-3.53%	1.08	1.07
2021/9	1757	1.53%	3.97%	3.63%	1.12	1.11
2021/10	1783.9	-0.41%	-0.06%	-0.17%	1.12	1.11
2021/11	1776.5	2.93%	2.37%	2.58%	1.15	1.13
2021/12	1828.6	-1.76%	-3.94%	-3.57%	1.10	1.09
2022/1	1796.4	5.81%	-2.04%	-0.53%	1.08	1.09
2022/2	1900.7	2.80%	0.52%	1.13%	1.08	1.10
2022/3	1954	-2.16%	-6.98%	-6.11%	1.01	1.03
2022/4	1911.7	-3.31%	0.25%	-0.48%	1.01	1.03

2022/5	1848.4	-2.22%	-5.38%	-4.90%	0.96	0.98
2022/6	1807.3	-1.41%	6.64%	5.16%	1.02	1.03
2022/7	1781.8	-3.12%	-4.10%	-3.91%	0.98	0.99
2022/8	1726.2	-2.39%	-7.50%	-6.57%	0.90	0.92
2022/9	1684.9	5.35%	4.20%	4.62%	0.94	0.97
2022/10	1775.1	3.78%	4.66%	4.52%	0.99	1.01
2022/11	1842.2	6.51%	-4.21%	-2.15%	0.95	0.99
2022/12	1962.2	-5.55%	5.30%	3.18%	1.00	1.02
2023/1	1853.2	8.14%	-2.96%	-0.72%	0.97	1.01
2023/2	2004.1	0.71%	3.59%	3.01%	1.00	1.04
2023/3	2018.3	-1.79%	1.21%	0.62%	1.01	1.05
2023/4	1982.1	-2.66%	-0.44%	-0.85%	1.01	1.04
2023/5	1929.4	4.14%	3.38%	3.69%	1.04	1.08
2023/6	2009.2	-2.16%	1.61%	0.93%	1.06	1.09
2023/7	1965.9	-4.09%	-1.36%	-1.93%	1.04	1.07
2023/8	1885.4	5.78%	-4.18%	-2.22%	1.00	1.04
2023/9	1994.3	2.20%	-2.85%	-1.87%	0.97	1.02
2023/10	2038.1	1.65%	8.02%	6.86%	1.05	1.09
2023/11	2071.8	-0.70%	4.04%	3.12%	1.09	1.13

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