The Application of Mathematical Optimization Theory in Economic Policy Design and Its Theoretical Discussion on The Efficiency of Resource Allocation

Jiayu Mei

College of Arts and Science, New York University, New York, NY 10003-2170, USA

Abstract. The application of mathematical optimization theory in economic policy design and its theoretical influence on the efficiency of resource allocation is the content that this paper wants to discuss. Firstly, this paper reviews the development context of mathematical optimization theory and its application foundation in economics, and focuses on the specific application of such optimization methods as linear programming, nonlinear programming and dynamic programming in economic policy making. Then explores how these optimization methods help policy makers in the complex economic environment to make more scientific and reasonable decision making and in the allocation of resources, industrial structure adjustment, the macroeconomic regulation and control, etc, especially for, The article also explores the mathematical optimization theory to improve the efficiency of resource allocation of the theoretical contributions including pareto optimality, mining, and further development of the concept of general equilibrium theory, this article also talked about mathematical optimization theory in the actual economic policy design challenges and limitations such as model assumes there is a deviation with actual situation, data acquisition and processing is difficult, Finally, this paper looks at the development trend of mathematical optimization theory in the future economic policy design, and focuses on the potential application prospect of emerging technologies such as big data and artificial intelligence. This research not only deepenes people's understanding of the application of mathematical optimization theory in economics, but also provides theoretical support for improving the scientific and effective economic policy.

Keywords: Mathematical optimization theory; Economic policy design; Efficiency of resource allocation; Decision science; Pareto optimality.

1. Introduction

An important branch of mathematics called mathematical optimization theory plays an increasingly important role in the field of economics. With the deepening of economic globalization and the increasing complexity of market environment, governments and economists around the world are facing the major challenge of how to formulate scientific and rational economic policies to achieve efficient allocation of resources. Under such circumstances, Mathematical optimization theory to economic policy design and assessment provides powerful tools and methods.

In recent years, the application of mathematical optimization theory in the design of economic policy increasingly, the international association for the study of economic policy (IEPR) statistics show that between 2018 and 2022, global mathematical optimization method for the research of economic policy analysis and design project number average growth rate of 15% a year, In the United States, the European Union, Japan and other developed countries and regions in the growth rate as high as 20%, a trend that shows the importance of mathematical optimization theory in modern economic policymaking more highlighted.

Mathematical optimization theory in economic policy design how the specific application and what it brings to the efficiency of resources allocation theory is the want content, discussed in this paper through the introduction of the basic concepts and methods of mathematical optimization theory and analyze it in fiscal, monetary, industry and trade in areas such as the application of policy optimization, This paper will explore how mathematical optimization theory can help policy makers make more scientific and reasonable decisions in complex economic environment. Moreover, this paper will also discuss the theoretical contribution of mathematical optimization theory to improving

the efficiency of resource allocation, including deepening and expanding the concepts of Pareto optimality and general equilibrium theory.

This study aims to provide a new perspective to deepen people's understanding of the application of mathematical optimization theory in economics and provide theoretical support for improving the scientific and effective economic policies. When the global economy is full of uncertainties, using mathematical optimization theory to optimize economic policy design will certainly provide important theoretical guidance and practical reference for countries to cope with economic challenges and achieve sustainable development.

2. Basic concepts and methods of mathematical optimization theory

2.1. Overview of mathematical optimization theory

Mathematical optimization theory belongs to a branch of mathematics, which mainly studies how to find the optimal solution when there are constraints. The core idea is to build a mathematical model and use specific algorithms and methods to find the solution that makes the objective function achieve the maximum or minimum value when meeting certain constraints. This theory has become an extremely powerful tool for resource allocation, decision analysis and policy making in the field of economics.

The general form of the mathematical optimization problem can be expressed as follows.

$$\text{minimize} f(x) \text{subject to} g_i(x) \leq 0, i = 1, \ldots, m h_j(x) = 0, j = 1, \ldots, p$$

Where, the objective function is f(x), the inequality constraint is g_i(x), the equality constraint is h_j(x), and the decision variable x is the parameter we need to optimize.

In the design of economic policy, the maximization of social welfare or the optimization of some economic indicators may be regarded as the objective function, while the budget constraint, resource constraint or other policy constraints may become the constraint conditions. Policy makers can find the optimal policy combination under the given conditions by solving this optimization problem.

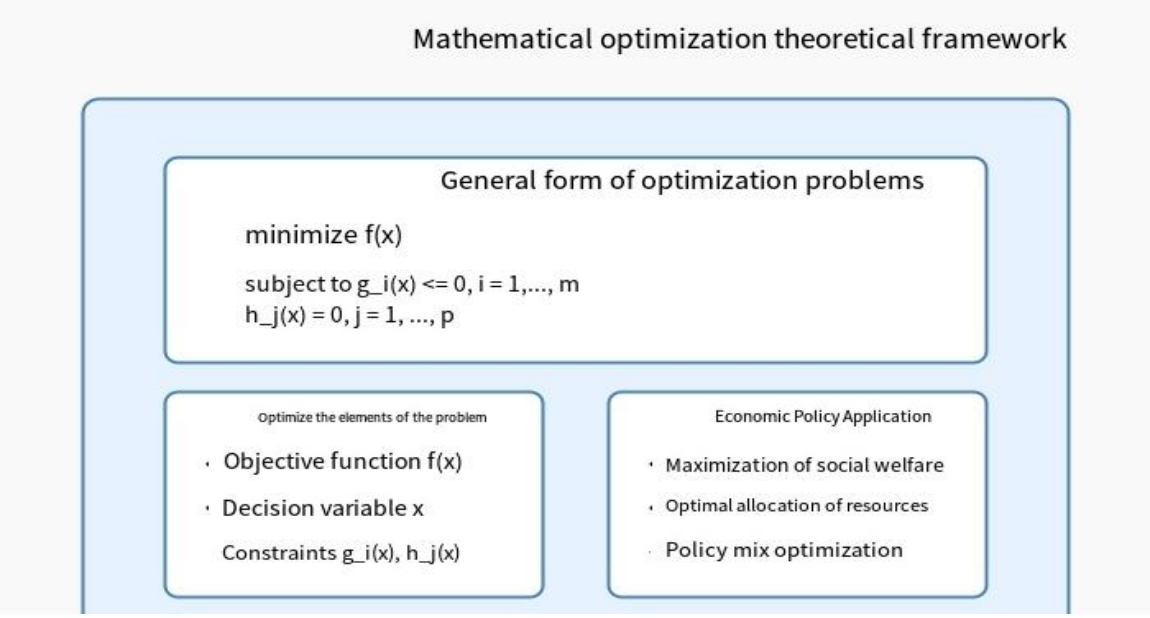

Figure 1 Basic framework and application of mathematical optimization theory

2.2. Introduction of main optimization methods

A variety of methods are included in mathematical optimization theory, and different types of optimization problems have their own applicable methods. The following are several optimization methods commonly used in economic policy design.

Table 1 Main optimization methods and their characteristics

Optimization methods	Apply the type of questions	Key Features	Typical applications
Linear programming	Linear objective function and constraints	It is computationally efficient and easy to implement	Resource allocation, production planning
Nonlinear programming	Nonlinear objective functions or constraints	It can handle more complex problems, but with high computational complexity	Portfolio optimization, economic equilibrium analysis
Dynamic programming	Multi-stage decision problems	Decompose complex problems into sub-problems and solve them step by step	Economic growth model, optimal control
Integer programming	Problems where the decision variables are integers	Suitable for discrete decision making, but difficult to solve	Facility location and project selection
Stochastic programming	Problems with random variables	Consider uncertainty and get closer to reality	Risk management, inventory control

These methods provide a variety of tools for economic policy design, so the most appropriate optimization method[1] can be selected according to the characteristics of the specific problem.

2.3. BApplication basis of mathematical optimization theory in economics

The application basis of mathematical optimization theory in economics is mainly reflected in the following aspects:

Theoretical modeling in economics provides an important tool. Many economic models, such as consumer utility maximization and manufacturer profit maximization, are essentially optimization problems. By abstracting economic behaviors into mathematical optimization models, economists can describe and analyze economic phenomena more accurately.

Secondly, the mathematical basis of economic equilibrium analysis is provided by optimization theory. Pareto optimal state in general equilibrium theory is essentially the solution of multi-objective optimization problem. With optimization theory, economists can further explore the nature and conditions of market equilibrium.

Thirdly, the optimization theory provides a quantitative method for policy evaluation. After constructing the optimization model, policy makers can simulate the effects of different policy programs to select the optimal policy mix, which is widely used in macroeconomic policy, industrial policy and other fields.

Finally, the economic prediction and decision making have the scientific basis provided by the optimization theory. Under the condition of uncertainty, methods such as stochastic optimization and robust optimization can help policy makers to formulate more robust economic policies.

In general, the development of economic research has been greatly promoted by the support of a set of systematic methodology provided by mathematical optimization theory, which plays a particularly important role in quantitative analysis and policy design.

3. The application of mathematical optimization theory in economic policy design

3.1. Optimization of fiscal policy

In the design of fiscal policy, mathematical optimization theory is mainly applied in the aspects of budget allocation, tax system design and public expenditure efficiency optimization. After

establishing the optimization model, policy makers can achieve the maximization of social welfare or achieve specific economic goals under the constraints of limited financial resources.

Taking budget allocation as an example, the government can construct a multi-objective optimization model that includes economic growth rate, employment rate, income distribution fairness and other indicators as the objective function, and the total budget limit and the minimum budget demand of each department as the constraints. By solving this optimization problem, the optimal budget allocation scheme under fixed conditions can be obtained.

According to the data of the World Bank, the number of countries using mathematical optimization method to design fiscal policy increased by 30% from 2018 to 2022, and the number of developing countries increased by 40%. It can be seen that the application of mathematical optimization method in fiscal policy design is rapidly spreading worldwide.

In the design of tax system, the optimization theory has an important influence. After building a dynamic optimization model that includes the impact of tax on economic behavior,[2] policy makers can design an optimal tax system that not only ensures fiscal revenue but also minimizes the distortionary effect of tax.

Figure 2 Application framework of mathematical optimization theory in fiscal policy design

3.2. Optimization of monetary policy

In the field of monetary policy, mathematical optimization theory is mainly used in interest rate decision, inflation target setting and foreign exchange reserve management. The central bank constructs a dynamic stochastic general equilibrium (DSGE) model and combines it with optimization algorithm to formulate the optimal monetary policy.

For interest rate decision, central banks can construct optimization models that cover inflation rate, output gap, interest rate smoothing and other factors. The objective function may be to minimize the weighted sum of squares of inflation rate and output gap, and the constraints include Phillips curve, IS curve and other macroeconomic relations. Solving this optimization problem can lead to the optimal level of interest rates under different economic conditions.

The international monetary fund (IMF), according to the statistics from 2020 to 2022, the world's major central Banks when monetary policy by using the mathematical optimization model of ratio rose to 75% from 60%, this shows that the optimization method in monetary policy design of importance.

In the management of foreign exchange reserves, the key role of optimization theory should not be underestimated. After building a multi-objective optimization model including risk, return, liquidity and other factors, the central bank can formulate the optimal foreign exchange reserve portfolio strategy, so as to achieve the effectiveness [6] of reserve asset management on the basis of ensuring the national economic and financial security.

3.3. Optimization of industrial policies

In terms of industrial policy design, mathematical optimization theory is mainly applied to industrial structure adjustment, resource allocation optimization, and coordinated development of industrial chains, etc. After constructing a general equilibrium model containing multiple industrial sectors and combining optimization algorithms, policy makers can design policy plans to promote coordinated development of industries and improve overall economic efficiency.

In the process of industrial structural adjustment, an optimization model can be constructed to achieve the maximum economic growth rate or employment rate, which is restricted by resource limitation and environmental protection requirements. The optimal industrial structure under given

conditions can be obtained by solving the model, thus providing a scientific basis [5] for the formulation of industrial policies.

According to the report of the Organization for Economic Cooperation and Development (OECD), from 2018 to 2022, the proportion of OECD member countries using mathematical optimization methods in industrial policy formulation increased from 45% to 65%. This trend shows that the scope of application of optimization methods in industrial policy design is expanding rapidly.

For resource configuration optimization, policymakers can design more efficient resource allocation mechanism with the aid of mathematical optimization theory, like building space equilibrium model containing different regions and industries to optimize regional industry layout to promote balanced development among regions, the development strategy of the region in recent years, this method is widely used and effective improve the pertinence and effectiveness of industrial policy.

3.4. Trade policy optimization

In the field of trade policy, mathematical optimization theory is mainly used in tariff structure design, trade agreement evaluation, and international market access strategy formulation. By constructing a multi-country and multi-sector computable general equilibrium (CGE) model, coupled with optimization algorithms, policy makers can design trade policies that maximize national interests.

As design tariff structure, can build up to maximize benefits for domestic and the world trade organization (WTO) rules, constrained optimization model of bilateral trade agreements, solve the model can get different trading conditions of the optimal tariff structure, thus to supply scientific basis for the establishment of trade policy.

The world trade organization (WTO) data show that during 2019 to 2022, the WTO member countries in trade policymaking process using mathematical optimization method of proportion rose to 55% from 70%, which reflects the application of optimization method in the design of the trade policy increasingly deepened[7].

Trade pact assessment, the importance of mathematical optimization theory to be reckoned with, and constructs a model of dynamic optimization containing multiple countries and departments, can simulate the effects of different trade agreement for national economy, and to provide trade negotiations and agreements signed decision support, regional trade agreement in nearly two years, this method is widely used, In recent two years, this method has been widely used, which effectively improves the scientific and forward-looking nature of trade policy.

4. Influence of mathematical optimization theory on the efficiency of resource allocation

4.1. Evaluation index of resource allocation efficiency

An important indicator of economic system operation is resource allocation efficiency, which can directly reflect the rationality of social resource utilization and the effectiveness of economic policies. In the field of economics, the following key indicators are often used in the evaluation of resource allocation efficiency:

The most basic evaluation criterion is Pareto efficiency, which refers to the state that the welfare level of no one can be further improved without harming the interests of no one. This concept provides a theoretical basis for judging whether the optimal resource allocation can be achieved.

Secondly, the efficiency with which a production unit uses input factors can be measured by technical efficiency, which is the ratio of the actual output to the maximum possible output under a given input.

In addition, allocative efficiency is measured by how well resources are allocated to different uses and by examining whether the marginal outputs of production factors are equal to ensure that resources are optimally allocated.

In addition, whether production units operate at the optimal scale can be reflected by scale efficiency, and scale efficiency is evaluated by comparing the actual scale with the optimal scale to evaluate[3] the efficiency of resource utilization.

Finally, dynamic efficiency focuses on the rationality of long-term resource allocation, covering many aspects such as technological progress, innovation ability and sustainable development, and evaluates the optimal degree of resource allocation in the process of long-term economic development.

Resource allocation efficiency can be evaluated by these indicators that evaluate it from different dimensions, so as to provide a comprehensive assessment tool for economic policy makers, so that they can analyze and improve resource allocation more accurately.

4.2. Positive impact of the application of optimization theory on resource allocation efficiency

The application of mathematical optimization theory in economic policy design has a significant positive effect on the improvement of resource allocation efficiency, because optimization theory first provides a scientific analysis framework for decision makers, so that they can make more rational and accurate decisions in complex economic environment. Just like linear programming model can help enterprises maximize profits when limited resources are constrained. And then improve production efficiency and resource utilization rate.

Secondly, the rational allocation of resources is promoted by the optimization theory. Once the multi-objective optimization model is established, policy makers can take into account the interests of multiple parties and find the best resource allocation scheme, which improves the overall economic efficiency and helps the realization of social equity.

Moreover, the predictability and adaptability of economic policies are enhanced by the application of optimization theory. Dynamic programming and other methods enable policy makers to formulate long-term strategies under uncertain conditions, thus improving the forward-looking and flexibility of policies, which is very important for coping with economic fluctuations and crises and conducive to maintaining economic stability and development.

In addition, the optimization and adjustment of economic structure is promoted by optimization theory. The construction of industrial structure optimization model can guide the formulation of industrial policies and promote the flow of resources to high-efficiency and high-value-added industries, so as to improve the overall economic efficiency.

Finally, the application of optimization theory can improve the accuracy of macroeconomic regulation, because after the establishment of macroeconomic models, policy makers can more accurately evaluate the impact of different policies, and then formulate more effective fiscal and monetary policies to optimize the allocation of resources to various economic sectors.

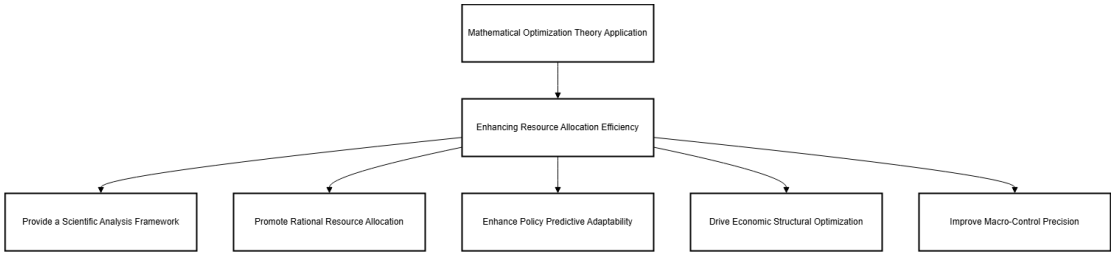

Figure 3 Positive effects of mathematical optimization theory on the efficiency of resource allocation

4.3. Limitations and potential negative effects of the application of optimization theory

It is true that mathematical optimization theory plays an important role in improving the efficiency of resource allocation, but its application is not without limitations and potential negative effects, because optimization models are often constructed based on a series of assumptions, which may be inconsistent with reality. For example, the assumption of perfect competition market cannot be fully consistent with reality, which may lead to a gap[8]

Secondly, high quality data input is the basis of the application of optimization theory, but in the actual economic environment, data acquisition and processing often encounter many challenges, such as data may be incomplete, inaccurate or time-delay, which makes the reliability of optimization results may be questioned.

Moreover, the economic system is complex and dynamic, and excessive reliance on mathematical models may be ignored, because economic activities are affected by many factors, such as psychological, cultural, social and other factors, which are difficult to quantify and model thoroughly. Therefore, if the emphasis on optimization is made, the decision makers may ignore the importance of these "soft factors".

In addition, short-term optimization may contradict long-term development goals, just as the single-minded pursuit of short-term efficiency may lead to over-exploitation of resources or damage to the environment, which will affect long-term sustainable development.

Finally, the inequality of resource distribution may be aggravated by the application of optimization theory, because although the optimization model can improve the overall efficiency, it may ignore the fairness of distribution, resulting in the accumulation of social resources in a few efficient sectors or groups, thus further widening the gap between the rich and the poor.

Recognizing these limitations and potential negative impacts will make us more prudent in applying optimization theory and adopt corresponding complementary measures in policy making.

4.4. Policy recommendations for improving resource allocation efficiency

After in-depth analysis of the application of mathematical optimization theory, in order to further improve the efficiency of resource allocation, this paper gives the following policy suggestions:

Firstly, the construction of data infrastructure should be strengthened. The government should invest in building a comprehensive, accurate and timely data collection and processing system to provide high-quality input data for the optimization model, and encourage data sharing and opening to promote the integration and application of cross-department and cross-field data.

In addition, it is necessary to promote the diversification and comprehensive development of the optimization model.

In order to reduce market failure, it is necessary to improve the market mechanism and create a fair and competitive market environment by relying on laws, regulations and institutional design to reduce information asymmetry, externalities and other problems, so that the market is closer to the ideal optimization state.

It is also important to strengthen talent training and technological innovation. It is necessary to cultivate interdisciplinary talents to improve the ability of decision-makers to apply optimization theory, and it is also necessary to encourage the innovation of optimization algorithms and models to improve their applicability and accuracy[4]

It is important to establish a dynamic evaluation and adjustment mechanism, regularly evaluate the implementation effect of optimization policies, and timely adjust the model parameters and assumptions according to the changes of economic environment to ensure that the policies are always effective.

These measures can better play the role of mathematical optimization theory in economic policy design and improve the efficiency of resource allocation to promote sustainable economic development.

5. Conclusion

The application of mathematical optimization theory in economic policy design and its impact on the efficiency of resource allocation are the contents of in-depth discussion in this study. After analyzing relevant theories and practices, the following conclusions are drawn:

Mathematical optimization theory brings powerful tools and methods to economic policy design. Optimization methods such as linear programming, nonlinear programming and dynamic

programming are of great significance in the fields of resource allocation, industrial structure adjustment and macroeconomic regulation and control. They can help decision-makers make more scientific and reasonable decisions in complex economic environment, thus greatly improving the efficiency of resource allocation.

Secondly, the efficiency of resource allocation is significantly improved by the application of optimization theory, which promotes the rational allocation of resources and makes economic policies more predictable and adaptable. It also promotes the adjustment of economic structure and makes macroeconomic regulation more accurate.

However, we should be aware of the limitations and potential negative effects of the application of optimization theory, such as the deviation of model assumptions from the actual situation, the problem of data quality, the excessive reliance on mathematical models to ignore the complexity of the economic system, the conflict between short-term optimization and long-term development goals, and the possibility of increasing the inequality of resource allocation, which should be paid attention to and solved in practical application.

Based on the above understanding, this paper proposes a series of policy suggestions to improve the efficiency of resource allocation, such as strengthening the construction of data infrastructure, promoting the diversification and integration of optimization models, improving the market mechanism, strengthening talent training and technological innovation, and building a dynamic evaluation and adjustment mechanism.

In general, although the mathematical optimization theory has a broad application prospect in economic policy design, it should be applied carefully and constantly innovated. With the development of emerging industrial technologies such as big data and artificial intelligence, the application of optimization theory in economic policy design will be more in-depth and extensive, and can bring more strong support to the improvement of resource allocation efficiency and sustainable economic development.

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