Research on crop planting optimization strategy based on MILP

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Abstract. North China is an important food production base in China, but its arable land types are diverse and scattered, optimizing crop planting strategies is conducive to the sustainable development of agriculture in the region. In this paper, we first preprocessed the data, then took the number of acres planted with crops as the decision variable, maximized the benefit as the objective function, determined the constraints from the aspects of succession planting, crop rotation, and convenience of field cultivation and management, and established a mixed-integer linear programming generalized model under the methods of lagging and half-price selling to determine the optimal planting plan of the countryside for the crops in the years of 2024 to 2030. Then, according to the relationship between each crop type and the cultivated land and the independence between the crop sales returns, the overall benefits are divided into three parts, solved by Lingo. Finally, the optimal benefits of each part are summed up to get the overall optimal benefits. The results of the study showed that the overall return from selling at half price was higher than the overall return from selling at stalled price, and for different types of crops, the overall return from selling at reduced price was higher for vegetable crops, rice, and edible mushroom crops, while the overall return from selling at stalled price was higher for the rest of the food crops, except for rice. The cultivation of edible mushroom crops, especially more potent morel mushrooms, which have higher yields and costs, can bring considerable benefits to the farmers of the village in the coming years.

Keywords: Crop Planting Strategy, MILP, Sustainable Development.

1. Introduction

The rural mountainous area of North China is vast, and mountain agriculture has an important position in national agriculture, however, the region has complex topography, limited arable land resources, and harsh climatic conditions, with low year-round temperatures and long crop growth cycles [1-2]. Therefore, it is important to optimize the planting strategy and adapt to the needs of agricultural production by using natural resources according to local conditions. In recent years, many scholars have conducted research on crop planting strategies, for example, Li Yanbin et al established a multi-objective agricultural planting structure optimization model based on the improved particle swarm algorithm by introducing inertia weight attenuation and particle variance strategy with the comprehensive objective of maximizing the benefits of economic, social, ecological and water resources [3]. Guo Jiajun et al analyzed the year-on-year growth changes of major crop yields in Wuxiang County from 2015 to 2019, and used the comprehensive comparative advantage index method, to conduct a comparative advantage analysis, to adjust the planting strategy [4]. Su Xiaofei used big data analytics to construct a crop planting decision model by collecting and analyzing data from various aspects to optimize agricultural production and resource allocation [5]. Although many scholars have conducted in-depth research in the optimization of crop planting strategies, the traditional planting methods in the face of natural and social factors make it difficult to make full use of land resources, and the effective use of crop planting spatial pattern and spatiotemporal dynamics, the use of natural resources by local conditions for the needs of agricultural production is very important [6]. This study deeply analyzes the spatio-temporal characteristics of the arable land and each crop in the region, as well as the unique constraint relationship that exists between the two, when solving the model, in order to reduce the difficulty of the traditional optimization method in solving the complex constraint logic, this paper divides the overall return into three independent modules, which are solved separately, and this chunking method reduces the complexity of the model solution, which is suitable for the collaborative optimization of the scenario of multiple types of crops. In addition, it also analyzes the impact of the two sales strategies of stagnant sales and half-price sales on crop revenue, which provides the basis for differentiated sales decision-making for the farmers in the area, and has good guiding significance.

2. Research methods

2.1. Data acquisition and preprocessing

The main source of data for the village is the official website of the National University Mathematical Modeling Competition (http://www.mcm.edu.cn). The type of arable land and crops grown in the village are as follows: There are 54 cultivated plots, including 34 open fields and 20 greenhouses. Open cultivated land includes flat dry land, terraced land, hillside land and watered land, of which flat dry land, terraced land and hillside land can only grow food crops in one season; watered land can grow rice in one season or vegetables in two seasons, and can only grow Chinese cabbage, white radish and carrot in the second season; greenhouse cultivation is divided into ordinary greenhouses and intelligent greenhouses, and the ordinary greenhouses can grow vegetables in one season and edible fungus in one season and edible fungus can only be grown in the second season; the intelligent greenhouses can grow vegetables in two seasons. Ordinary greenhouses can grow one season of vegetables and one season of edible fungus, and edible fungus can only be grown in the second season; intelligent greenhouses can grow two seasons of vegetables. In addition to this, the official website also provides us with data on the area under different crops, cost of cultivation, acreage, and sale price of the village in 2023.

The main objective of this study is to maximize the agricultural income of the village in the coming years, and for subsequent modeling, the following information was again obtained by searching for relevant information: According to the latest information released by the agricultural departments of crops grown in the mountainous regions of North China and the relevant industry associations (official website of the Ministry of Agriculture and Rural Development: “Agricultural Products Market Monitoring Report of North China” released), the stagnation rate of crops was obtained to be around 5%; the expected future sales volume of each crop remained stable relative to 2023 (database of the National Bureau of Statistics: annual sequence of sales volume of sub-crops of North China from 2018 to 2023), from which 95% of the total crop production in 2023 (5% stunting rate) was used to represent the expected sales volume [7-8]. Finally, the available data were preprocessed for subsequent modeling.

2.2. Introduction to the methodology

Mixed Integer Linear Programming (MILP) is a mathematical modeling approach for optimization problems [9]. In MILP, the objective function and constraints are linear and the problem contains both continuous and integer variables. By considering the minimization or maximization of a certain metric in the objective function while restricting the range of values of the variables in the constraints, MILP can help to find the optimal decision-making solution. The method is widely used in production scheduling and resource allocation and is no exception in agricultural development, e.g., Zhang, L., & Xu, C. Combining 0-1 variables and continuous variables to construct a MILP model of coupled systems to solve the rural energy transition problem [10]. Li, J., & Chen, W. Enhancing food supply chain resilience through dynamic programming to deal with time dimension (seasonal planting decision) and combining linear programming to optimize spatial layout [11].

3. Modeling and Solving

3.1. Preparation for modeling

Analyzing the crop planting situation, we found that there is a certain correlation between crop types and planting arable land, in order to make this correlation more clear, for this reason, we drew a Wayne diagram between crop types and planting arable land, as shown in Figure 1 below.

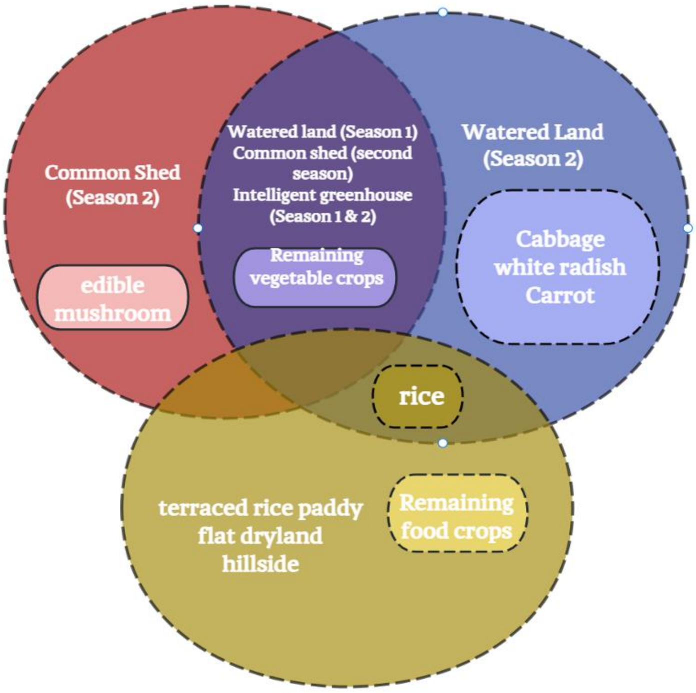

Figure 1. Wayne diagrams between species types and cultivated cropland

From Figure 1 above, it can be seen that the relationship between crop types and cultivated land is more complex, the remaining food crops (except rice) are only planted in the flat dry land, terraces, and slopes in a single season, and there is no intersection with other crop types and cultivated land, so the benefits of the food crops do not affect the overall benefits; edible fungi are only planted in the second season in the ordinary greenhouse, and there is no intersection with other crop types and cultivated land; while for the remaining crops, the relationship between each crop type and cultivated land is complicated, and splitting the calculation can only achieve partial optimization but not the overall optimization, so the benefits are calculated as a whole. For the remaining crops, the relationship between each crop type and planting arable land is intricate and complex, which can only achieve partial optimization but not overall optimization, so it will be regarded as a whole to calculate the benefits, which can be seen that there is revenue independence between the three parts of the planting division, and therefore its profit can be calculated separately when calculating the overall benefit maximization, i.e., as long as the above three parts of the overall benefit maximization. The maximization of their respective interests, the sum of the three interests is the maximization of the overall interest. The specific classification is shown in Table 1 below:

Table.1. Classification of different crop types

Split Categories	Crop Types
Part I	Remaining food crops
Part II	Edible mushroom
Part III	Rice, remaining vegetable crops, cabbage - carrot - white radish

Because the subsequent model involves a lot of numbered data, to facilitate the understanding of this study, the crops and plots were numbered, and by the crops and plots of the number of the alternation to represent the alternation of the year: there are a total of 59 crops in the countryside, so that i denotes the number of the crops, i = 1...59, then when i = 60...118 then represents the second year, and so on. Similarly, there are 54 plots in the village, let j denote the number of crops, j = 1...54, then when j = 55...108 then it represents the second year.

Considering the specificity of certain crops, special attention needs to be paid to 16 (rice), 35-37 (cabbage-carrot-white radish), and 38-41 (edible mushrooms). And for special legume crops, this paper notates the legume array $u[i']$=[1,2,3,4,5, 17, 18, 19], where i'=0..7 . The legume cropland array $v[j']$=[1,2..54] where j'=0..53.

3.2. Generalized Modeling of Revenue Maximization

(1) Establishment of the objective function

Since the magnitude of the expected sales of a crop about the total production of the season cannot be determined, the objective function is considered in segments. This is shown in equation (1) below:

$$\max W=\begin{cases}\sum_{i=1}^{7n} p_i\cdot S_i-\sum_{i=1}^{7n}\sum_{j=1}^{7m} C_{ij}\cdot x_{ij}\cdot a_{ij}, & P_i>S_i\\ \sum_{i=1}^{7n}\sum_{j=1}^{7m} p_i\cdot P_i-\sum_{i=1}^{7n}\sum_{j=1}^{7m} C_{ij}\cdot x_{ij}\cdot a_{ij}, & 0\le P_i\le S_i\end{cases} \tag{1}$$

where i is the number of the crop, i = 1...7n, at this time n = 59; S_i is the expected sales volume of the i^{th} crop; j is the number of the plot, j = 1...7m, at this time m = 54; C_{ij} is the cost of planting the i^{th} crop in the j^{th} plot; W is the total return in the next 6 years when stagnation is used; x_{ij} is the number of acres planted by the i^{th} crop in the j^{th} plot; p_i is the selling unit price of the i^{th} crop; P_i is the total production of the i^{th} crop in a year, a_{ij} is a 0-1 variable, as shown in equation (2) below:

$$a_{ij}=\begin{cases}1, & \text{The i}^{\text{th}}\text{ crop is planted on the j}^{\text{th}}\text{ plot}\\ 0, & \text{The i}^{\text{th}}\text{ crop is not planted on the j}^{\text{th}}\text{ plot}\end{cases} \tag{2}$$

The half-price sale is similar to the objective function of the stagnant sale and is still considered segmented. This is shown in equation (3) below:

$$\max W^*=\begin{cases}\sum_{i=1}^{7n} p_i\cdot S_i-\sum_{i=1}^{7n}\sum_{j=1}^{7m} C_{ij}\cdot x_{ij}\cdot a_{ij}+0.5\sum_{i=1}^{7n} p_i\cdot\left(P_i-S_i\right), & P_i>S_i\\ \sum_{i=1}^{7n} p_i\cdot P_i-\sum_{i=1}^{7n}\sum_{j=1}^{7m} C_{ij}\cdot x_{ij}\cdot a_{ij}, & 0\le P_i\le S_i\end{cases} \tag{3}$$

where W^* is the total return over the next six years when half-price sale is used, and the meanings of the remaining variables are consistent with equations (1) and (2), and will not be repeated here.

(2) Establishment of constraints

Successive cropping constraints

Growing the same crop on the same plot for a long period results in soil elemental imbalance, deterioration of physico-chemical properties, and vulnerability of the crop to a single pest or disease. Therefore, there is a need to regulate continuous cropping.

$$a_{ij}+a_{i+n,j+m}\le 1,\quad i=1..6n,\ j=1..6m \tag{4}$$

From equation (2), a_{ij} indicates whether the i^{th} crop is planted on the j^{th} plot or not, and equation (4) is expressed as not planting the same crop in two consecutive years.

Crop rotation constraints

Crop rotation is the regular rotation of different crops on the same piece of land for the purpose of equalizing soil nutrients [14]. The present study focuses on constraining this situation by planting legume crops at least once in every three years in the village.

$$a_{u[i']+nk,v[j']+mk}+a_{u[i']+n(k+1),v[j']+m(k+1)}+a_{u[i']+n(k+2),v[j']+m(k+2)}\geq 1 \tag{5}$$

According to the previous section, $u[i']$ is the legume array, $v[j']$ is the legume cropland array, and k is noted as the year iteration variable. Where $i'=0\ldots7$, $j'=0\ldots53$, and $k=0\ldots5$.

Farming and management constraints

The following constraints have been established because farmers generally do not allow crops to be spread out too thinly or individual plots (including greenhouses) to be too small in order to facilitate farming operations and field management:

First, for rice #16, if one season of rice is chosen to be planted on a watered plot, the plot is to be planted with all rice, at which point the number of acres with rice planted on the j^{th} plot is equal to the acreage planted on that plot.

$$x_{16+nk,j}=m_j\cdot a_{16+nk,j} \tag{6}$$

where $x_{16+nk,j}$ denotes the number of acres of rice planted on the j^{th} plot in the kth year, m_j denotes the acreage planted on the j^{th} plot, and $a_{16+nk,j}$ denotes whether or not rice was planted on the j^{th} plot in the kth year. $j=1\ldots7m$, $k=1\ldots7$.

Second, for all legume crops, if legumes are selected for planting in plot j, then all legumes are to be planted in that plot, at which point the number of acres with legumes planted in plot j equals the acreage of that plot.

$$x_{u[i']+nk,j}=m_j\cdot a_{u[i']+nk,j} \tag{7}$$

The interpretation of Eq. (7) is similar to that of Eq. (6) except that it is replaced by legume crops. $i'=0\ldots7$, $j=1\ldots7m$, $k=1\ldots7$.

Finally, for other types of crops, it was found that a maximum of two crops were being co-cropped in the same plot. There are:

$$x_{ij}\geq\frac{m_j}{2},\quad i\neq 16,u\left[i'\right] \tag{8}$$

where x_{ij} denotes the number of acres of the i^{th} crop planted in the j^{th} plot and m_j denotes the number of acres in the j^{th} plot.

Watered land planting constraints

From the previously mentioned cultivation law in the countryside: the watered land can be planted with one season of rice or two seasons of vegetables, and the second season can only be planted with cabbages, white radishes, and carrots, which needs to be considered separately here due to the "or" relationship between crops planted in the watered land:

$$\sum_{j=27k}^{34k}a_{16j}\cdot a_{ij}=0,\quad i=17k..34k,\quad k=1..7 \tag{9}$$

Watered land is numbered 27 to 34, and other vegetable crops that can be grown on watered land are numbered 17 to 34.

Productive capacity constraints

The production capacity of a cropland is related to the area of that cropland, and the decision variable in this model is x_{ij}, i.e., the number of acres of the i^{th} crop planted on the j^{th} plot, such that the maximum acreage planted is M acres, and the minimum acreage planted is m acres, and thus there is:

$$m \le x_{ij} \le M, \quad i = 1..7n,\ j = 1..7m \tag{10}$$

(3) Model aggregation

It is clear that in the models for both treatments, only the objective function differs, while the constraints are common to both. The models are summarized below:

Adoption of the stagnant sales approach:

$$\max W = \begin{cases} \sum_{i=1}^{7n} p_i \cdot S_i - \sum_{i=1}^{7n}\sum_{j=1}^{7m} C_{ij} \cdot x_{ij} \cdot a_{ij}, & P_i > S_i \\ \sum_{i=1}^{7n}\sum_{j=1}^{7m} p_i \cdot P_i - \sum_{i=1}^{7n}\sum_{j=1}^{7m} C_{ij} \cdot x_{ij} \cdot a_{ij}, & 0 \le P_i \le S_i \end{cases} \tag{11}$$

Adoption of the price reductions approach:

$$\max W^* = \begin{cases} \sum_{i=1}^{7n} p_i \cdot S_i - \sum_{i=1}^{7n}\sum_{j=1}^{7m} C_{ij} \cdot x_{ij} \cdot a_{ij} + 0.5\sum_{i=1}^{7n} p_i \cdot \left(P_i - S_i\right), & P_i > S_i \\ \sum_{i=1}^{7n} p_i \cdot P_i - \sum_{i=1}^{7n}\sum_{j=1}^{7m} C_{ij} \cdot x_{ij} \cdot a_{ij}, & 0 \le P_i \le S_i \end{cases} \tag{12}$$

Constraints:

$$\text{s.t.} \begin{cases} x_{16+nk,j} = m_j \cdot a_{16+nk,j} \\ x_{u\left[i'\right]+nk,j} = m_j \cdot a_{u\left[ii'\right]+nk,j} \\ x_{ij} \ge \dfrac{m_j}{2}, \quad i \ne 16, u\left[i'\right] \\ \sum_{j=27k}^{34k} a_{16j} \cdot a_{ij} = 0, \quad i = 17k..34k, k = 1..7 \\ m \le x_{ij} \le M, \quad i = 1..7n,\ j = 1..7m \\ a_{ij} + a_{i+n,j+m} \le 1, \quad i = 1..6n,\ j = 1..6m \\ a_{u\left[i'\right]+nk,v\left[j'\right]+mk} + a_{u\left[i'\right]+n(k+1),v\left[j'\right]+m(k+1)} + a_{u\left[i'\right]+n(k+2),v\left[j'\right]+m(k+2)} \ge 1 \end{cases} \tag{13}$$

3.3. Establishment of revenue maximization chunking optimization model

3.3.1 Adoption of stagnation methods (part I)

(1) Establishment of the objective function

As can be seen from Table 1, the first part is the remaining food crops. There are a total of 15 different types of crops, i.e., n = 15; and a total of 26 different plots, i.e., m = 26. Therefore, the following adjustments are made on top of equation (1):

$$\max W_1 = \begin{cases} \sum_{i=1}^{7\times15} p_i \cdot S_i - \sum_{i=1}^{7\times15}\sum_{j=1}^{7\times26} C_{ij} \cdot x_{ij} \cdot a_{ij}, & P_i > S_i \\ \sum_{i=1}^{7\times15}\sum_{j=1}^{7\times26} p_i \cdot P_i - \sum_{i=1}^{7\times15}\sum_{j=1}^{7\times26} C_{ij} \cdot x_{ij} \cdot a_{ij}, & 0 \le P_i \le S_i \end{cases} \tag{14}$$

(2) Establishment of constraints

Collaborative constraints

Substituting n=15, m=26 into Eq. (4).

$$a_{ij} + a_{i+15,j+26} \le 1, \quad i = 1..15\times6,\ j = 1..26\times6 \tag{15}$$

Bean rotation constraints

Substituting n=15, m=26 into Eq. (5).

$$a_{u[i']+15k,v[j']+26k}+a_{u[i']+15(k+1),v[j']+26(k+1)}+a_{u[i']+15(k+2),v[j']+26(k+2)}\geq 1 \tag{16}$$

In this question $u[i']$ is still the legume array and $v[j']$ is still the legume cropland array, but at this point $i'=0\ldots4$ and $j'=0\ldots25$, i.e., only beans in grain are taken. Remember that k is the year iteration variable, k=0..5.

Farming and management constraints

Substituting n=15, m=26 into Eq. (7), (8), where i'=0...4, j=1...26, k=1…7.

For legume crops:

$$x_{u[i']+15k,j}=m_j\cdot a_{u[i']+15k,j} \tag{17}$$

For other types of crops:

$$x_{ij}\geq\frac{m_j}{2},\quad i\neq u[i'] \tag{18}$$

Productive capacity constraints

The largest acreage in the first part is 80 acres and the smallest acreage is 13 acres, so Eq. (10) is adjusted to:

$$13\leq x_{ij}\leq 80 \tag{19}$$

(3) Model aggregation

$$\max W_1=\begin{cases}\sum_{i=1}^{7\times15}p_i\cdot S_i-\sum_{i=1}^{7\times15}\sum_{j=1}^{7\times26}C_{ij}\cdot x_{ij}\cdot a_{ij}, & P_i>S_i\\ \sum_{i=1}^{7\times15}\sum_{j=1}^{7\times26}p_i\cdot P_i-\sum_{i=1}^{7\times15}\sum_{j=1}^{7\times26}C_{ij}\cdot x_{ij}\cdot a_{ij}, & 0\leq P_i\leq S_i\end{cases} \tag{20}$$

$$\text{s.t.}\begin{cases}a_{ij}+a_{i+15,j+26}\leq 1,\quad i=1..15\times6,\ j=1..26\times6\\ x_{u[i']+15k,j}=m_j\cdot a_{u[i']+15k,j}\\ x_{ij}\geq\dfrac{m_j}{2},\quad i\neq u[i']\\ 13\leq x_{ij}\leq 80\\ a_{u[i']+15k,v[j']+26k}+a_{u[i']+15(k+1),v[j']+26(k+1)}+a_{u[i']+15(k+2),v[j']+26(k+2)}\geq 1\end{cases} \tag{21}$$

3.3.2 Adoption of stagnation methods (part II)

(1) Establishment of the objective function

From Table 1, the second part is edible mushrooms. There are a total of 4 different types of crops, i.e., n = 4; and there are a total of 16 different plots of roots, i.e., m = 16. The following adjustments were made on top of equation (1):

$$\max W_2=\begin{cases}\sum_{i=1}^{7\times4}p_i\cdot S_i-\sum_{i=1}^{7\times4}\sum_{j=1}^{7\times16}C_{ij}\cdot x_{ij}\cdot a_{ij}, & P_i>S_i\\ \sum_{i=1}^{7\times4}\sum_{j=1}^{7\times16}p_i\cdot P_i-\sum_{i=1}^{7\times4}\sum_{j=1}^{7\times16}C_{ij}\cdot x_{ij}\cdot a_{ij}, & 0\leq P_i\leq S_i\end{cases} \tag{22}$$

(2) Establishment of constraints

Farming and management constraints

In solving this part of the problem, it is no longer necessary to constrain the rice and bean crops because only one crop, edible mushroom, is considered. Equation (8) is amended to:

$$x_{ij} \geq \frac{m_j}{2} \tag{23}$$

Productive capacity constraints

Since edible mushrooms can only be grown in ordinary greenhouses, all of which have a planting area of 0.6 acres, there:

$$0 \leq x_{ij} \leq 0.6 \tag{24}$$

3.3.3 Adoption of stagnation methods (part III)

(1) Establishment of the objective function

As shown in Table 1, the third component includes rice, remaining vegetable crops, and cabbage-white radish-carrot. There are a total of 39 different types of crops for roots, i.e., n = 39; and a total of 16 different plots, i.e., m = 28. The following adjustments are made on top of equation (1):

$$\max W_3 = \begin{cases} \sum_{i=1}^{7\times39} p_i \cdot S_i - \sum_{i=1}^{7\times39}\sum_{j=1}^{7\times28} C_{ij}\cdot x_{ij}\cdot a_{ij}, & P_i > S_i \\ \sum_{i=1}^{7\times39}\sum_{j=1}^{7\times28} p_i \cdot P_i - \sum_{i=1}^{7\times39}\sum_{j=1}^{7\times28} C_{ij}\cdot x_{ij}\cdot a_{ij}, & 0 \leq P_i \leq S_i \end{cases} \tag{25}$$

(2) Establishment of constraints

Successive cropping constraints

Since only cabbage (19), white radish (20) and red radish (21) can be grown in the second season in the watered land, there is a need to constrain the succession of these three vegetables.

$$a_{ij} + a_{i+39,\, j+28} \leq 1, \quad i = 1..39\times6,\ j = 1..28\times6 \tag{26}$$

Crop rotation constraints

$$a_{u[i']+39k,\, v[j']+28k} + a_{u[i']+39(k+1),\, v[j']+28(k+1)} + a_{u[i']+39(k+2),\, v[j']+28(k+2)} \geq 1 \tag{27}$$

In this question $u[i']$ is still the legume array and $v[j']$ is still the legume cropland array, but at this point i' = 5...7 and j' = 25...53, i.e., only beans from vegetables are taken. Remember that k is the year iteration variable, k=0...5.

Farming and management constraints

For legume crops, where i'=5...7, j=1...28, k=1…7.

$$x_{u[i']+39k,\, j} = m_j \cdot a_{u[i']+39k,\, j} \tag{28}$$

For other types of crops:

$$x_{ij} \geq \frac{m_j}{2}, \quad i \neq u[i'] \tag{29}$$

Watered land planting constraints

$$\sum_{j=17k}^{34k} a_{16j} \cdot a_{ij} = 0, \quad i = 17k..34k, \quad k = 1..7 \tag{30}$$

Productive capacity constraints

$$0.6 \leq x_{ij} \leq 22 \tag{31}$$

(3) Model aggregation

$$\max W_3 = \begin{cases} \sum_{i=1}^{7\times39} p_i \cdot S_i - \sum_{i=1}^{7\times39}\sum_{j=1}^{7\times28} C_{ij}\cdot x_{ij}\cdot a_{ij}, & P_i > S_i \\ \sum_{i=1}^{7\times39}\sum_{j=1}^{7\times28} p_i \cdot P_i - \sum_{i=1}^{7\times39}\sum_{j=1}^{7\times28} C_{ij}\cdot x_{ij}\cdot a_{ij}, & 0 \le P_i \le S_i \end{cases} \tag{32}$$

$$\text{s.t.} \begin{cases} a_{ij} + a_{i+39,j+28} \le 1, \quad i = 1..39\times6,\ j = 1..28\times6 \\ x_{u\left[i'+39k,j\right.} = m_j \cdot a_{u\left[i'\right]+39k,j} \\ \sum_{j=17k}^{34k} a_{16j}\cdot a_{ij} = 0, \quad i = 17k.34k, \quad k = 1..7 \\ 0.6 \le x_{ij} \le 22 \\ x_{ij} \ge \dfrac{m_j}{2}, \quad i \ne u\left[i'\right] \\ a_{u\left[i'\right]+39k,v\left[j'\right]+28k} + a_{u\left[i'\right]+39(k+1),\left[j'j'+28(k+1)\right)} + a_{u\left[i'\right]+39(k+2),v\left[j'\right]+28(k+2)} \ge 1 \end{cases} \tag{33}$$

3.3.4 Adoption of the half-price method of sale (three-part summary)

Due to space constraints and the fact that the constraints for each of the subsections are the same for the approach using half-price selling as they are for the approach using stagflation, only a summary of the model for the three sections is given here.

(1) Part I

$$\max W_1^* = \begin{cases} \sum_{i=1}^{7\times15} p_i \cdot S_i - \sum_{i=1}^{7\times15}\sum_{j=1}^{7\times26} C_{ij}\cdot x_{ij}\cdot a_{ij} + 0.5\sum_{i=1}^{7\times15} p_i\cdot\left(P_i - S_i\right), & P_i > S_i \\ \sum_{i=1}^{7\times15} p_i \cdot P_i - \sum_{i=1}^{7\times15}\sum_{j=1}^{7\times26} C_{ij}\cdot x_{ij}\cdot a_{ij}, & 0 \le P_i \le S_i \end{cases} \tag{34}$$

$$\text{s.t.} \begin{cases} a_{ij} + a_{i+15,j+26} \le 1, \quad i = 1..15\times6,\ j = 1..26\times6 \\ x_{u\left[i'\right]+15k,j} = m_j \cdot a_{u\left[i'\right]+15k,j} \\ x_{ij} \ge \dfrac{m_j}{2}, \quad i \ne u\left[i'\right] \\ 13 \le x_{ij} \le 80 \\ a_{u\left[i'\right]+15k,v\left[j'\right]+26k} + a_{u\left[i'\right]+15(k+1),v\left[j'\right]+26(k+1)} + a_{u\left[i'\right]+15(k+2),v\left[j'\right]+26(k+2)} \ge 1 \end{cases} \tag{35}$$

(2)Part II

$$\max W_2^* = \begin{cases} \sum_{i=1}^{7\times4} p_i \cdot S_i - \sum_{i=1}^{7\times4}\sum_{j=1}^{7\times16} C_{ij}\cdot x_{ij}\cdot a_{ij} + 0.5\sum_{i=1}^{7\times4} p_i\cdot\left(P_i - S_i\right), & P_i > S_i \\ \sum_{i=1}^{7\times4} p_i \cdot P_i - \sum_{i=1}^{7\times4}\sum_{j=1}^{7\times16} C_{ij}\cdot x_{ij}\cdot a_{ij}, & 0 \le P_i \le S_i \end{cases} \tag{36}$$

$$\text{s.t.} \begin{cases} x_{ij} \ge \dfrac{m_j}{2} \\ 0 \le x_{ij} \le 0.6 \end{cases} \tag{37}$$

(3) Part III

$$\max W_3^* = \begin{cases} \sum_{i=1}^{39} p_i \cdot S_i - \sum_{i=1}^{39}\sum_{j=1}^{28} C_{ij} \cdot x_{ij} \cdot a_{ij} + 0.5\sum_{i=1}^{39} p_i \cdot \left(P_i - S_i\right), \quad P_i > S_i \\ \sum_{i=1}^{39} p_i \cdot P_i - \sum_{i=1}^{39}\sum_{j=1}^{28} C_{ij} \cdot x_{ij} \cdot a_{ij}, \quad 0 \le P_i \le S_i \end{cases} \tag{38}$$

$$\text{s.t.} \begin{cases} a_{ij} + a_{i+39,\, j+28} \le 1, \quad i = 1..39\times 6,\, j = 1..28\times 6 \\ x_{u[i']+39k,\, j} = m_j \cdot a_{u[i']+39k,\, j} \\ \sum_{j=17k}^{34k} a_{16j} \cdot a_{ij} = 0, \quad i = 17k..34k, \quad k = 1.7 \\ 0.6 \le x_{ij} \le 22 \\ x_{ij} \ge \frac{m_j}{2}, \quad i \ne u\left[i'\right] \\ a_{u[i']+39k,\, v[j']+28k} + a_{u[i']+39(k+1),\, v[j']+28(k+1)} + a_{u[i']+39(k+2),\, v[j']+28(k+2)} \ge 1 \end{cases} \tag{39}$$

3.4. Analysis of results

Difference in revenue between the two sales methods

Based on the results of Lingo's solution, it can be concluded that the overall return from selling at half price is higher than the overall return from selling at stalled prices, and for different types of crops, the overall return from vegetable crops, rice, and edible fungi crops is high by carrying out sales at reduced prices, whereas the return from the rest of the food crops except rice is slightly higher by selling at stalled prices than by selling at reduced prices.

Differences in cropping structure between the two sales methods

Taking edible mushroom crops as an example, by visualizing the results of solving the problem for edible mushroom crops in both the slow-selling and half-price selling scenarios, the differences in the cropping structure under these two sales methods for the goal of maximizing returns are further analyzed. Figure 2-a below shows the annual cultivation of edible mushrooms in the case of stagnant sales, and Figure 2-b below shows the annual cultivation of edible mushrooms in the case of half-price sales.

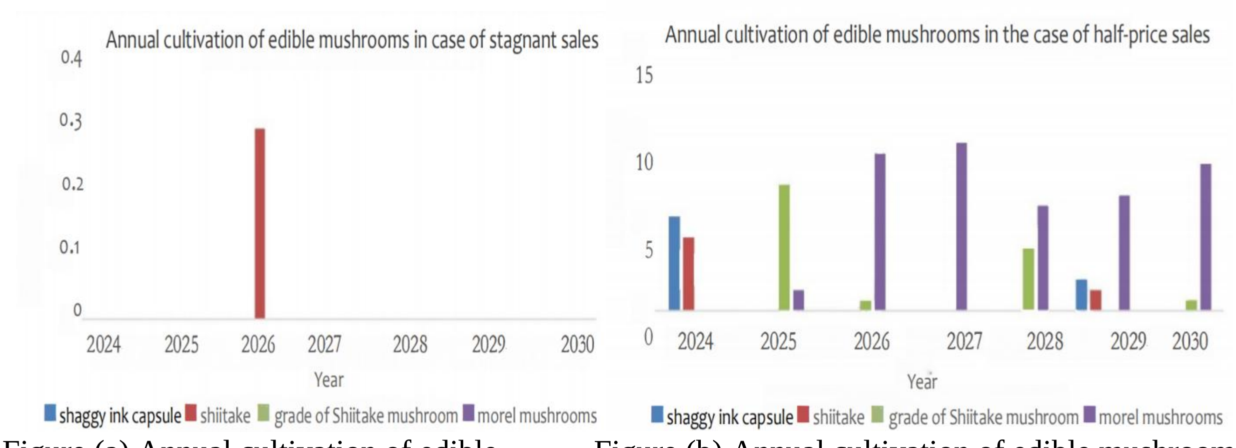

Figure (a) Annual cultivation of edible mushrooms in case of stunting

Figure (b) Annual cultivation of edible mushrooms under the half-price sale case

Figure 2. Annual cultivation of edible mushrooms under two modes of sale

Comparing the above results, it can be found that: in the case of stagnant sales, only 0.3 acres of shiitake mushrooms were chosen to be planted in 2026, while they were not planted at other times;

in the case of half-price sales, almost every type of edible mushrooms was planted, and moreover, morels were planted for six consecutive years. As the mu yield and planting cost of edible mushrooms are very high, i.e., edible mushrooms are very easy to have the situation that the total production exceeds the expected sales volume, and if the exceeding part is directly handled in accordance with the stagflation, this will bring huge losses for farmers, so it is not suitable to plant edible mushrooms with large mu yield and high cost in the stagflation situation. And the total production exceeds the expected sales of part of the half-price way to sell, because the sale of edible mushrooms unit price is high, even if the price is lowered after the sale of edible mushrooms may also bring considerable gains for farmers; according to known data, the sale of morel mushrooms unit price of 80 ~ 120 yuan, much higher than other edible mushrooms, so there are a few years of planting morel mushrooms in successive years.

4. Conclusions

Based on the arable land status, crop cultivation structure, and the sales price and mu yield of each crop in a mountainous area in North China, this study establishes an optimization model of cultivation strategy under two different selling methods by making full use of the spatial pattern of crop cultivation and the dynamic changes in space and time, considering the constraints according to the local conditions, and taking the maximization of benefits as the objective function. It was concluded that selling at a reduced price could bring higher income to the village, and it was not appropriate to sell edible mushrooms with high mu yield and planting cost at a stagnant price. The model still has shortcomings, in the modeling process, it does not consider the influence of external factors such as natural disasters, extreme weather, policies and the internal relationship between crops on the model, in the follow-up work, on the one hand, the model can be re-solved by adding the relevant external factors as constraints, on the other hand, it can be supplemented with the analysis of the substitutability and complementarity between crops, and the introduction of correlation coefficients to the model to be correction to further improve the practicality and objectivity of the model.

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