Planting Scheme Design Method Based on Linear Programming Model

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Abstract. To optimize precision agriculture implementation and foster the sustainable growth of rural economies, this study proposes a method for designing crop planting schemes based on a linear programming model. First, a linear programming model is developed to define decision variables that meet specified constraints. Two objective functions are established: surplus yield beyond sales remains unsold, and surplus yield is sold at half price. Second, the model incorporates five key constraints: land area limitations, plot constraints, crop adaptability, continuous cropping restrictions, and other relevant factors. Third, based on existing farmland conditions, crop characteristics in a rural village in the North China mountainous region, and statistical data on rural crop planting in 2023, data preprocessing is performed. The model is then solved through simulation verification. The results yield two planting schemes and their corresponding profits for the next decade. Under the first scheme, the projected average total profit from 2024 to 2030 is 6,052,184.19 CNY, while the second scheme yields an average total profit of 8,564,127.26 CNY over the same period. Ultimately, the optimal planting scheme is identified, ensuring maximum profitability while considering the natural conditions of plots and crops.

Keywords: Linear programming model; crop planting plan; sustainable development; rural agriculture.

1. Introduction

Effectively utilizing limited arable land and promoting organic farming in rural areas is crucial for sustainable economic development. This study examines a rural village in North China, considering factors such as land characteristics, seasonal constraints, crop rotation, and yield reductions due to continuous cropping. A scientifically sound and adaptable planting strategy is developed to optimize land use, enhance agricultural productivity, mitigate operational risks, and promote long-term economic sustainability.

Existing research indicates that this village has six types of land available for cultivation: dry flatland, terraced fields, hillside land, irrigated land, conventional greenhouses, and smart greenhouses. Greenhouse planting areas are typically 0.3 or 0.6 hectares. While dry flatland, terraced fields, and hillside land are suitable for single-season crops, irrigated land supports either one season of rice or two vegetable-growing cycles per year. Conventional greenhouses can accommodate two different crop rotations annually, whereas smart greenhouses are designed for cultivating two cycles of the same crop each year [1,2,3].

Strategic agricultural planning plays a pivotal role in determining farmers' income levels. Previous studies have typically designed planting schemes based on factors such as crop planting dates, climate conditions, and soil properties [4,5]. For instance, Sacks W. J. (2010) analyzed the relationships between planting dates, temperature, precipitation, and potential evapotranspiration using 30-year average climatologies from the Climatic Research Unit at the University of East Anglia (CRU CL 2.0) [6]. These studies investigated the spatial and temporal suitability of various crops under different climatic conditions, as well as the role of soil moisture in cultivation planning.

In addition to natural factors, socioeconomic constraints also influence crop selection and planting strategies [7,8]. Some researchers have developed agricultural planning models that account for limited resource availability—such as water, labor, fertilizers, and seeds—aiming to optimize resource allocation, reduce input costs, and enhance productivity to maximize net revenue [9].

Most of these studies primarily consider the impact of natural conditions on crop cultivation. However, within the same region, factors such as climate, rainfall, minor topographical variations, and soil conditions alone do not ultimately determine which crops should be planted [10,11].

Therefore, this study takes both the sales performance of various crops and natural conditions into account while also incorporating socioeconomic factors such as market demand and farmers' income. The objective is to maximize farmers' earnings while meeting market demand to the greatest extent possible. From this perspective, data were collected for a rural village in 2023, including soil types, land area by soil type, the suitability of different crop types for various soils and their respective planting seasons, crop yields per unit area, sales prices, and production costs. Based on this dataset, a linear programming model was developed to formulate two distinct crop planting schemes for different land plots under varying strategic considerations.

2. Development of the Linear Programming Model for the Planting Scheme

2.1. Objective Function

Under the constraints of limited planting area, different types of land use, varying crops for different seasons on different plots, restrictions on continuous cropping on the same plot, and the requirement that each plot must grow legumes at least once every three years, this study will develop an objective planning model to determine the optimal crop planting scheme in order to achieve the highest possible profit.

The profit from crop cultivation is calculated by multiplying the crop’s sales volume by the sales price and subtracting the planting costs. If the total yield of a crop in a given season exceeds the expected sales volume, the surplus cannot be sold. This study proposes two methods for handling the excess yield beyond the expected sales volume and develops separate optimization models for each of these two approaches.

Option 1: Surplus yield remains unsold.

$$\max z^t = \sum_k \sum_i (p_i {s_i}^{t_k} - \sum_j c_i {x_{ij}}^{t_k}) \tag{1}$$

Option 2: Surplus yield is sold at 50% of the previous year's sales price.

$$\max z^t = \sum_k \sum_i [p_i {s_i}^{t_k} + 0.5 p_i \max(\sum_j {x_{ij}}^{t_k} y_i - s_i, 0) - \sum_j c_i {x_{ij}}^{t_k}] \tag{2}$$

In which, k represents the k^{th} season of each year, i denotes the index of the crop, j denotes the index of the plot, z^t represents the total profit from all crops harvested in year t, p_i represents the sales price of crop i, c_i represents the planting cost of crop i, ${x_{ij}}^{t_k}$ represents the planting area of crop i on plot j in season k of year t, s_i represents the expected sales volume of crop i, y_i represents the yield of crop i.

$${s_i}^{t_k} = \begin{cases} \sum_j {x_{ij}}^{t_k} y_i & \sum_j {x_{ij}}^{t_k} y_i < s_i \\ s_i & \sum_j {x_{ij}}^{t_k} y_i > s_i \end{cases} \tag{3}$$

${s_i}^{t_k}$ represents the sales volume of crop i sold at the normal price in season k of year t, which is the minimum of the predicted sales volume and the actual production volume.

The optimization modeling process for crop cultivation plans is depicted in Figure 1.

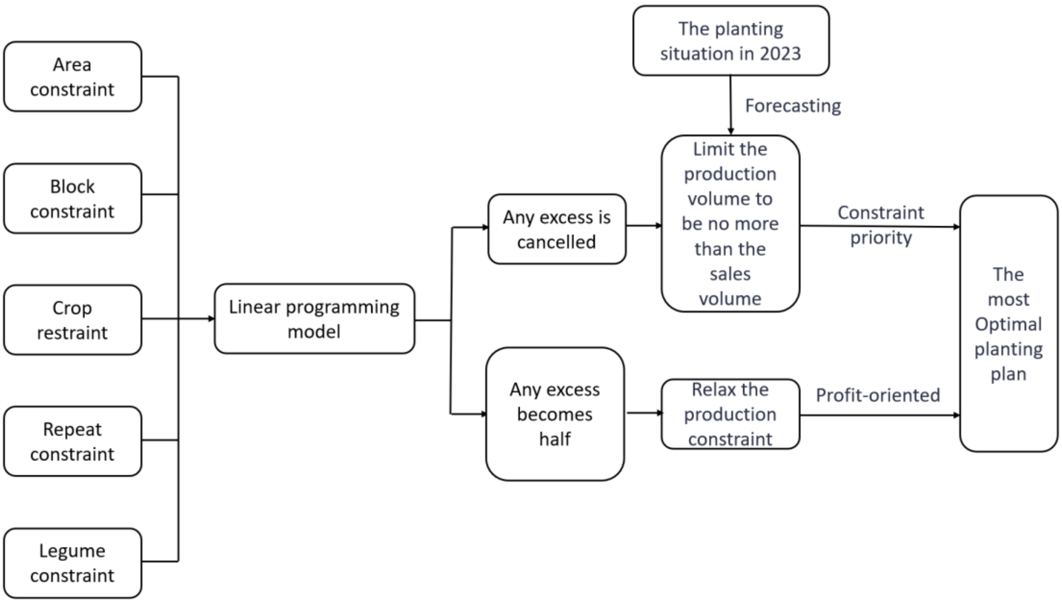

Figure 1. Conceptual Framework for the Development of the Crop Planting Scheme Model

2.2. Decision Variables

The decision variable is the planting area of crop i on plot j in season k of year t, denoted as ${x_{ij}}^{t_k}$.

2.3. Constraints Condition

2.3.1. Area Constraint

(1) The total planting area of all crops on plot j in season k of year t must not exceed the total area of that plot.

$$\sum_{i} {x_{ij}}^{t_k} \leq d_j \tag{4}$$

where ${x_{ij}}^{t_k}$ represents the planting area of crop i on plot j in season k of year t, and d_j represents the total area of plot j.

(2) To facilitate farming operations and field management, the planting area of each crop on a single plot should not be too small. It is assumed that the planting area for each crop on a given plot must not be less than the area planted in the previous year.

$${x_{ij}}^{t_k} \geq \min {x_{ij}}^{t-1_k} \tag{5}$$

where the planting area for crops in greenhouses is either 0.3 or 0.6, while the planting area in non-greenhouse plots must be an integer.

2.3.2. Single-Season and Double-Season Constraints for Plots

(1) Dry flatland, terraced fields, and hillside land can only be used for one crop per year, meaning that in the second season, the planting area of all crops in these three types of land is zero.

$$\sum_{i} \sum_{j \in A_1} {x_{ij}}^{t_2} = 0 \tag{6}$$

were,$a_1 = \{\text{Dry Flatland}\}. a_2 = \{\text{Terraced Fields}\}. a_3 = \{\text{Hillside Land}\}$。$A_1 = \{a_1. a_2. a_3\}$

(2) Rice is a single-season crop, meaning that in the second season, the planting area of rice on all plots is zero.

$$\sum_{j} x_{i_1 j}{}^{t_2} = 0 \tag{7}$$

where i_1 is the crop index corresponding to rice.

2.3.3. Crop Suitability Constraints

(1) Dry flatland, terraced fields, and hillside land are suitable for planting one crop per year.

$$\sum_{i} \sum_{j \in A_1} x_{ij}{}^{t_1} = \sum_{i \in B_{A_1}} \sum_{j \in A_1} x_{ij}{}^{t_1} = \sum_{j \in A_1} d_j \tag{8}$$

where, B_{A_1} represents the set of crops suitable for planting on the plots in $A_1 = \{a_1.a_2.a_3\}$.

(2) Irrigated land is suitable for planting either one season of rice or two seasons of vegetables per year, with the total planting area for vegetables in both seasons being equal.

$$\sum_{j \in a_4} \sum_{i} x_{ij}{}^{t_1} = \sum_{j \in a_4} \sum_{i \in B_{a_4}^1} x_{ij}{}^{t_1} + \sum_{j \in a_4} x_{i_1 j}{}^{t_1} = \sum_{j \in a_4} d_j \tag{9}$$

$$\sum_{j \in a_4} \sum_{i \in B_{a_4}^2} x_{ij}{}^{t_2} = \sum_{j \in a_4} \sum_{i \in B_{a_4}^1} x_{ij}{}^{t_1} \tag{10}$$

where, $a_4 = \{Irrigated\ land\}$, i_1 is the crop index corresponding to rice, $B_{a_4}^1$ represents the vegetables suitable for planting on plot a_4 in the first season, and $B_{a_4}^2$ represents the vegetables suitable for planting on irrigated plot a_4 in the second season.

(3) Conventional greenhouses are suitable for planting two different crops in two seasons each year.

$$\sum_{j \in a_5} \sum_{i} x_{ij}{}^{t_1} = \sum_{j \in a_5} \sum_{i \in B_{a_5}^1} x_{ij}{}^{t_1} = \sum_{j \in a_5} d_j \tag{11}$$

$$\sum_{j \in a_5} \sum_{i} x_{ij}{}^{t_2} = \sum_{j \in a_5} \sum_{i \in B_{a_5}^2} x_{ij}{}^{t_2} = \sum_{j \in a_5} d_j \tag{12}$$

where, $a_5 = \{Conventional\ greenhouse\}$, $B_{a_5}^1$ represents the crops suitable for planting on plot a_5 in the first season, and $B_{a_5}^2$ represents the crops suitable for planting on plot a_5 in the second season.

(4) Smart greenhouses are suitable for planting the same crop in two seasons each year.

$$\sum_{j \in a_6} \sum_{i} x_{ij}{}^{t_1} = \sum_{j \in a_6} \sum_{i \in B_{a_6}} x_{ij}{}^{t_1} = \sum_{j \in a_6} d_j \tag{13}$$

$$\sum_{j \in a_6} \sum_{i} x_{ij}{}^{t_2} = \sum_{j \in a_6} \sum_{i \in B_{a_6}} x_{ij}{}^{t_2} = \sum_{j \in a_6} d_j \tag{14}$$

where, $a_6 = \{Smart\ greenhouse\}$, B_{a_6} represents the crops suitable for planting on plot a_6 in both seasons.

2.3.4. Each crop cannot be planted consecutively on the same plot in successive seasons

For crops planted in a single season, the sum of the planting areas of crop i on plot j in season k of year t and season k of year $t+1$ should equal the larger planting area of the two years. This implies that the planting area for the other year must be zero, thus satisfying the condition that if crop i is planted on plot j in one year, it cannot be planted on the same plot in the adjacent year.

For single season crops,

$$x_{ij}{}^{t_k} + x_{ij}{}^{t_k+1} = max(x_{ij}{}^{t_k}, x_{ij}{}^{t_k+1}) \tag{15}$$

For double season crops,

$$x_{ij}{}^{t_1} + x_{ij}{}^{t_2} = max(x_{ij}{}^{t_1}, x_{ij}{}^{t_2}) \tag{16}$$

2.3.5. A legume crop must be planted at least once every three years

$$\sum_{i \in K} \sum_{t_0=0}^{2} B_{ij}{}^{t+t_0} \geq 1 \tag{17}$$

where, $K = \{\text{legume crop}\}$, $B_{ij}{}^{t}$ is defined as:

$$B_{ij}{}^{t} = \begin{cases} 0 & x_{ij}{}^{t_1} + x_{ij}{}^{t_2} = 0 \\ 1 & x_{ij}{}^{t_1} + x_{ij}{}^{t_2} > 0 \end{cases} \tag{18}$$

This indicates whether a legume crop is planted on plot j in year t.

3. Simulation Validation

3.1. Data Processing

This study collects data on the existing farmland and crop conditions of a rural village in the mountainous region of North China, along with the 2023 rural crop planting and related statistical data. Based on the established optimization model, the optimal planting plan for the years 2024–2030 is provided.

First, Python was used to preprocess the 2023 statistical data. It was determined that each crop has a corresponding yield per mu, planting cost, and upper and lower limits of the selling price on different plots. To examine whether any missing or abnormal values exist in the yield per mu across different plots, a standard deviation test was conducted. A mean test was applied to the yield per mu, planting cost, and selling price for all crops except rice, Chinese cabbage, white radish, red radish, elm mushroom, shiitake mushroom, white beech mushroom, and morel mushroom. The results indicate that no abnormal values were found in the upper and lower limits of yield per mu, planting cost, and selling price for these crops.

The average values of yield per mu, planting cost, and the upper and lower limits of selling prices were calculated for each crop across different plots. These averages were then used as the representative yield per mu, planting cost, and selling price for each crop to simplify the program and reduce model complexity. The obtained basic data table is shown in Table 1.

Table. 1. Basic Data Table of Crops

Crop type	Number	Crop name	Yield per mu	Farmgate price (yuan/Jin)	Planting cost (yuan/mu)
Grain (beans)	1	Soybean	380	3.25	400.00
Grain (beans)	2	Black soya bean	475	7.50	400.00
Grain (beans)	3	Ormosia	380	8.25	350.00
⋮	⋮	⋮	⋮	⋮	⋮
Edible mushrooms	39	Mushroom	4000	19.00	2000.00
Edible mushrooms	40	Pleurotus nebrodensis	10000	16.00	10000.00
Edible mushrooms	41	Toadstool	1000	100.00	10000.00

Using MATLAB, box plots were generated for the selling price, planting cost, and yield per mu of all crops. The abnormal data patterns are depicted in Figures 2, 3 and 4.

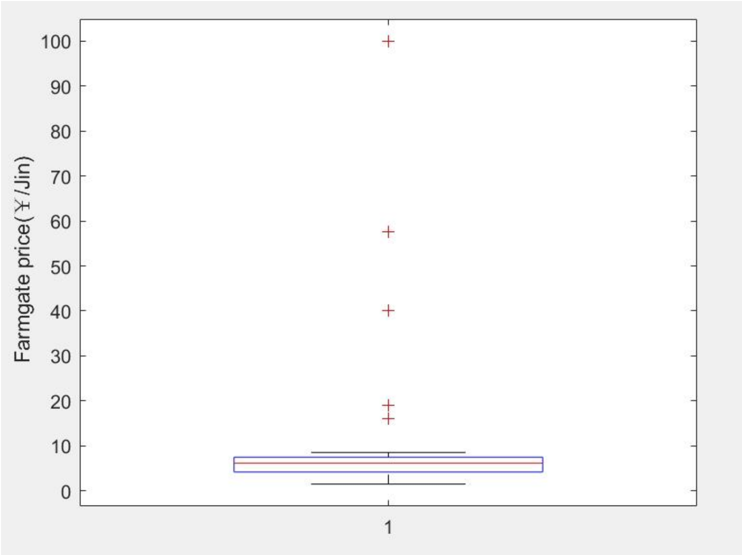

Figure 2. Box plot of Farmgate price(yuan/Jin) for all crops.

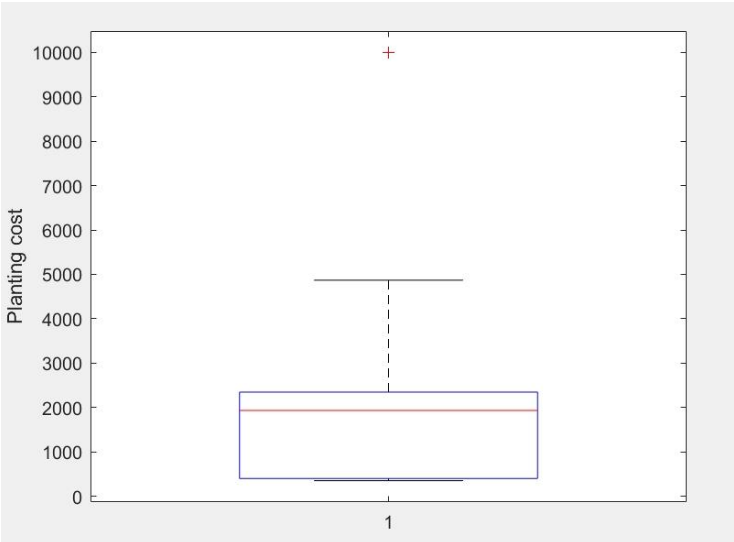

Figure 3. Box plot of planting cost for all crops.

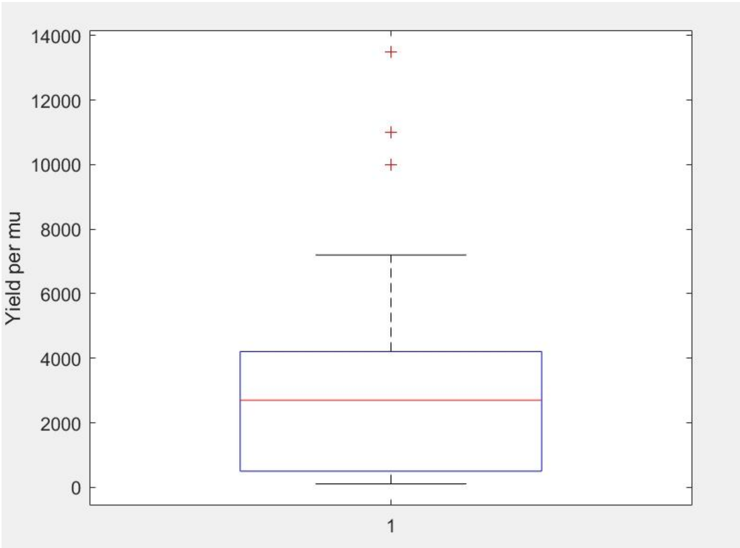

Figure 4. Box plot of yield per mu for all crops.

The results of the identified outliers were visualized, as shown below:

Table. 2. Results of outliers for each crop

Crop name	Per mu yield	Farmgate price (yuan/Jin)	Planting cost (yuan/mu)
Buckwheat	—	40.00	—
Cucumber	13500	—	—
Water spinach	11000	—	—
Elm mushroom	—	57.50	—
Mushroom	—	19.00	—
Pleurotus Nebrodensis	10000	16.00	10000.00
Toadstool	—	100.00	10000.00

Although these data points were identified as outliers, they align with real-world agricultural conditions. For instance, the high yield per mu of crops such as cucumbers, water spinach, and white beech mushrooms is due to their rapid growth rate and efficient land use, which maximizes space utilization. Similarly, crops like buckwheat and morel mushrooms exhibit high selling prices. Taking amorel mushrooms as an example, their rarity and high market value contribute to their elevated price. Therefore, in subsequent analyses, these data points will be considered as valid and included in the calculations.

3.2. Model Solution

The Python-based model solutions for unsold excess and discounted excess scenarios are visualized in Table. 3 and 4 (partial crop results displayed).

Table. 3. Optimal planting options for 2024 over unmarketable parts

Planting plot	Crop name	Planting area/mu	Planting season
A1	Ormosia	24	Single season
	Glutinous broom corn	25	Single season
	Buckwheat	14	Single season
	Sweet potato	17	Single season
A2	Black soya bean	17	Single season
	Millet	38	Single season
⋮	⋮	⋮	⋮
F4	Pepper	0.6	First season
	Tomato	0.3	Second season
	Green pepper	0.3	Second season

Table. 4. Optimal planting options in 2024 for the excess portion sold at half last year's price

Planting plot	Crop name	Planting area/mu	Planting season
A1	Ormosia	42	Single season
	Glutinous broom corn	15	Single season
	Buckwheat	15	Single season
	Sorghum	8	Single season
A2	Sweet potato	20	Single season
	Pumpkin	13	Single season
	Sorghum	12	Single season
	Glutinous broom corn	10	Single season
⋮	⋮	⋮	⋮
F4	Cucumber	0.6	First season
	Water spinach	0.6	Second season

By calculating the optimal planting results obtained above, the total profits for each year from 2024 to 2030 were determined. The comparison between multi-year average profits (both schemes) and the 2023 benchmark is visualized in Figure 5

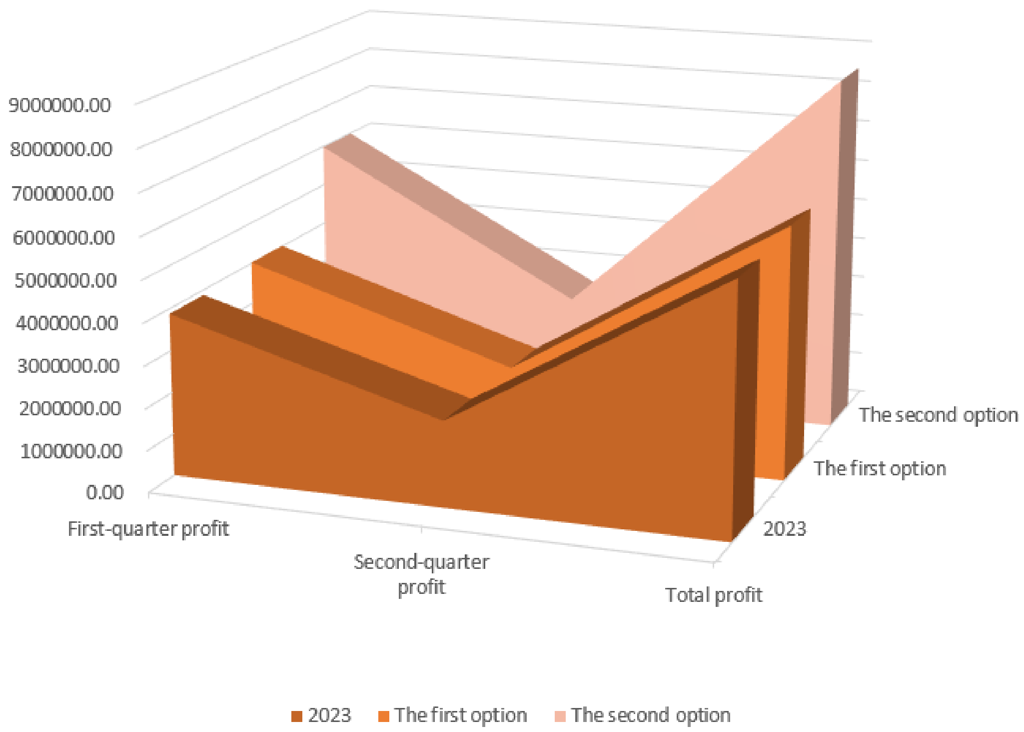

Figure 5. Annual average profit under two scenarios.

In conclusion, the scenario where the surplus is sold at 50% of last year's price yields higher profits than the scenario where the surplus remains unsold. In the first scenario, the average total profit from 2024 to 2030 is 6,052,184.187 yuan, with the highest total profit of 6,073,593.14 yuan in 2030. In the second scenario, the average total profit from 2024 to 2030 is 8,564,127.262 yuan, with the highest total profit of 10,190,727.83 yuan in 2024.

4. Conclusions

This study developed an optimal planting plan for a rural village in North China using a linear programming model, which considers various factors such as land characteristics, seasonal constraints, crop rotation, and market demand. The model's goal was to maximize profits while adhering to environmental, economic, and operational constraints.

The results indicate that the scenario where surplus crops are sold at 50% of the previous year's price yields higher profits compared to the scenario where surplus crops are unsold. Specifically, the first scenario resulted in an average total profit of 6,052,184.19 yuan from 2024 to 2030, with the highest profit of 6,073,593.14 yuan in 2030. In contrast, the second scenario, which involved selling surplus crops at a discount, resulted in a higher average total profit of 8,564,127.26 yuan, with the highest profit of 10,190,727.83 yuan in 2024.

These findings suggest that adopting a pricing strategy for surplus crops can significantly enhance profitability, thereby contributing to the sustainable development of rural economies by optimizing both agricultural production and marketing strategies.

In future research, the application of the PSO (Particle Swarm Optimization) model could further improve the model's efficiency and profitability. This study presents a ten-year planting plan based solely on data from a rural village in North China. Future studies will explore the potential to enhance profitability by applying ant colony optimization and genetic algorithms in more diverse natural conditions and demand scenarios. Finally, we hope that the planting scheme design approach presented in this study can be more widely applied to the development of planting schemes under various conditions.

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