Decision-Making Spare Parts Assembly Production Process Based on Dynamic Programming Algorithm

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Abstract. When enterprises produce best-selling electronic products, their production decisions have a crucial impact on cost control and profitability. This study focuses on the sampling and testing of spare parts and decision-making at each stage of the production process, and adopts statistical hypothesis testing and efficacy analysis to derive the key data: when the defective rate of spare parts has a nominal value of 10%, and the defective rate is deemed to have exceeded the nominal value and rejected at 95% confidence, the minimum number of samples tested is 133 and the number of rejected defective products is 19; when the defective rate is deemed to have not exceeded the nominal value and received at 90% confidence, the minimum number of samples tested is 81 and the number of received defective products is 4. In addition, we set up 0 - 1 decision variables to analyse the cost and profit function, and obtain the optimal decision and maximum profit in each case after exhaustive calculation, such as in case 1, when we test parts 1 and 2, do not test the finished products and do not dismantle the unqualified products, the profit can reach to 12,420. The decision-making model constructed in the study is accurate and practical, which helps enterprises to make scientific decisions and improve economic efficiency.

Keywords: Dynamic Programming, Production Decision Model, Parts Assembly.

1. Introduction

In the highly competitive electronics manufacturing industry, corporate production decisions are a matter of competitiveness. In the process of producing best-selling electronic products, from the procurement of spare parts to the delivery of finished products in various links, the decision-making is reasonable or not directly affect the cost and profit. For example, the procurement stage needs to accurately verify the nominal value of the defective rate provided by the supplier, and the sampling and testing programme affects the cost of testing and product quality [1]; in the production process, the decision-making on the testing of spare parts and finished products, the dismantling of unqualified finished products, and the treatment of unqualified products returned by the user are all closely related to the direct and indirect costs of the enterprise. Therefore, it is of great significance for enterprises to seek scientific production decision-making methods.

Research has been done to achieve certain results in the field of production decision-making. Some scholars use traditional optimisation algorithms, such as linear programming to deal with production resource allocation, which improves production efficiency [2]; some scholars have also achieved certain results with machine learning technology, like neural networks for product quality prediction and control [3]. However, traditional optimisation algorithms are computationally complex and difficult to obtain the global optimal solution when dealing with multi-stage and multi-constraint decision-making in complex production systems [4]. Although machine learning methods have high prediction accuracy, they rely heavily on data and do not fully take into account the comprehensive optimisation of cost and profit [5].

In view of this, this paper constructs a production decision-making model, using hypothesis testing, 0 - 1 planning and dynamic planning methods [6]. Through the hypothesis testing to design sampling and testing programs to reduce costs; 0 - 1 planning for discrete decision-making modelling to analyze the cost of profit; with the help of dynamic programming algorithms to solve multi-stage decision-making problems, to obtain the global optimal decision-making, to provide decision-making support for the enterprise, to help enterprises in the complex market environment to achieve cost control and profit maximization.

2. Design sampling and testing programmes with as few tests as possible

2.1. Modelling assumptions

1. It is assumed that firms are rational, i.e., they will make decisions at all stages with the objective of maximising profits and only profit maximisation as a goal.

2. It is assumed that the enterprise will purchase the same number of individual initial parts to facilitate modelling and simplification.

3. It is assumed that spare parts, semi-finished goods and finished goods held by the enterprise will not otherwise be missing or depreciated.

4. It is assumed that the values of defective production rates, inspection costs, dismantling costs, etc. will not fluctuate over time.

5. It is assumed that the enterprise is able to sell all the finished goods produced, i.e., the enterprise's revenue can be expressed as the product of the number of finished goods and the selling price of the finished goods.

6. It is assumed that finished products identified by testing as defective will be disassembled into components and returned to the sample pile retaining all of their properties.

2.2. Hypothesis modelling for sample testing

In this paper, we test the hypothesis model statistically, and design the optimal sampling and testing scheme through the method of efficacy analysis. By reviewing the related literature, it is known that the distribution of defective parts and accessories obeys the binomial distribution, i.e., X~ Bin (n, p), where n is the number of samples, p is the defective rate, and X is the number of defective parts [7].

Formulate hypotheses:

The test required is that the parameter value of the aggregate from which the sample is taken is greater or less than the given value, and it has been verified by a review of the relevant literature that a one-sided test is used in both cases [8].

First scenario:

Original hypothesis H0: p= 10%

Alternative hypothesis H0: p> 10% Significance α: 5 per cent

Second scenario:

Original hypothesis H0: p= 10%

Alternative hypothesis H0: p< 10% Significance α: 10%

2.3. Sample size determination and derivation of test decision conditions

Determining the sample size: given the confidence level, this paper uses two-sided hypothesis testing to calculate the rejection and acceptance domains [9].

The rate of substandard products detected in the sample was:

$$\hat{p} = \frac{X}{n} \tag{1}$$

When the sample size is large enough, the binomial distribution is approximated using the normal distribution according to the central limit theorem, $\mathrm{i.e.}, X \sim N\,(np, np(1-p))$.

Scenario 1: Reject the hypothesis and reject the shipment of parts when the rate of defects detected in the sample is sufficiently large at 95 per cent confidence. The standardised variable is:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}} \tag{2}$$

The corresponding critical value at the level of significance is $Z1 - \alpha = 1.645$. The condition for rejecting the original hypothesis is:

$$Z \geq Z1 - \alpha \tag{3}$$

assume (office)

$$\hat{p} \geq \mu + z_{1-\alpha} \frac{\sigma}{\sqrt{n}} \tag{4}$$

Through efficacy analysis, based on the minimum sample size formula [10]:

$$n = \frac{\left(z_\alpha \cdot \sqrt{p_0(1 - p_0)} + z_\beta \cdot \sqrt{p_1(1 - p_1)}\right)^2}{(p_1 - p_0)^2} \tag{5}$$

Here, assuming that the actual reject rate is p1= 0.2, the minimum number of tests is 133 95% confidence, and the number of rejects required to reject the original hypothesis is 19.

Scenario 2: Reject the original hypothesis and accept the batch of spare parts when the rate of rejections detected from the sample is sufficiently low at 90 per cent confidence. The standardised variables are:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}} \tag{6}$$

The corresponding critical value at the level of significance is Z1-α= 1.28. The condition for rejecting the original hypothesis is:

$$Z \leq Z1 - \alpha \tag{7}$$

assume (office)

$$\hat{p} \leq \mu - z_{1-\alpha} \frac{\sigma}{\sqrt{n}} \tag{8}$$

Through efficacy analysis, based on the minimum sample size formula:

$$n = \frac{\left(z_\alpha \cdot \sqrt{p_0(1 - p_0)} + z_\beta \cdot \sqrt{p_1(1 - p_1)}\right)^2}{(p_1 - p_0)^2} \tag{9}$$

Here, assuming that the actual reject rate is p1= 0.2, the minimum number of tests is 81 with 90% confidence, and the number of rejects required to reject the original hypothesis is 4.

3. Quantitative analysis and optimal solution search for multi-stage decision-making in production processes

Firstly, for each decision in the production process, 0-1 variables are set up, where X1 = 1 denotes the inspection of part 1 and X1= 0 vice versa; X2= 1 denotes the inspection of part 2, and X2 = 0 vice versa; X3 = 1 denotes the inspection of finished products, and X3= 0 vice versa; and X4= 1 denotes the dismantling of detected unqualified finished products, and X4 = 0 vice versa. and vice versa.

Each decision involves a certain cost and determines the final gain or loss of the business. The core objective of minimising the total cost is accomplished by analysing the profit function of the enterprise to optimise the decision making for each part [11]. Assuming that part 1.

2 and the cumulative number of finished products are n1, n2 and n3, respectively, and the defective rates of parts 1, 2 and finished products are p1, p2 and p3, respectively.

The cost incurred during the period of testing the parts is:

$$C1 = n1\,B1 + n2\,B2 + X1\,n1\,Z1 + X2\,n2\,Z2 \tag{10}$$

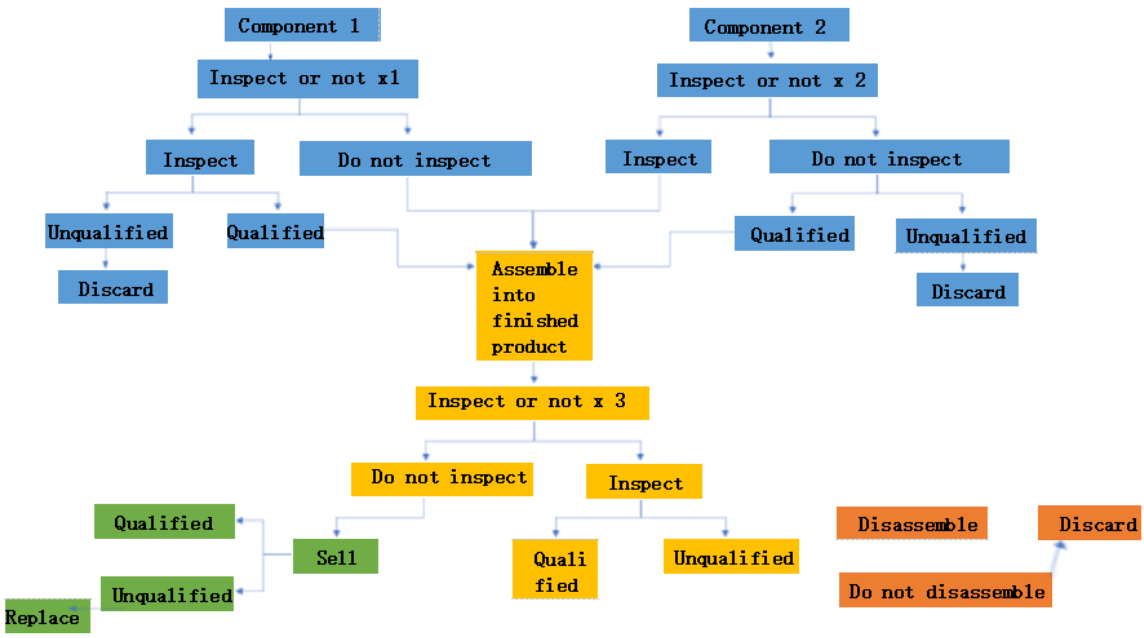

Figure 1. Schematic Diagram of Each Stage of Parts Production

Where B1 is the purchase unit price of Part 1; B2 is the purchase unit price of Part 2; Z1 is the cost of Part 1; and Z2 is the inspection cost of Part 2.

The cost incurred during the period of testing the finished product is:

$$C2 = n3\,B3 + X3\,n3\,Z3 \tag{11}$$

Where B3 is the assembly cost of the finished product and Z3 is the inspection cost of part 3.

Also note in this paper that all parts that are not detected as defective are used in the production of the finished product regardless of whether they are detected or not, so the cumulative number of finished products is:

$$n3 = \min\{(1 - X1p1)n1, (1 - X2p2)n2\} \tag{12}$$

Cumulative rate of defective finished products:

$$p = 1 - (1 - (1 - X1)p1)(1 - (1 - X2)p2)(1 - p3) \tag{13}$$

The costs incurred for the reconciliation are:

$$C3 = (1 - X3)n3pB4 \tag{14}$$

B4 is a loss on exchange.

In summary, the expectation of the objective function X4 = 0, i.e., without disassembling the detected substandard finished products, is as follows:

If there is a dismantling process, the cost incurred in the dismantling process is: n3pZ4, where Z4

$$\text{max profit } = n_3(1-p)\times \text{price} - \sum_{i=1}^{3} C_i \quad (15)$$

is the dismantling cost. At the same time, the parts generated by dismantling will incur an additional cost in the previous link, which is:

$$C_4 = X_3 X_4 \left(n_3 p Z_4 + X_1 n'_1 Z_1 + X_2 n'_2 Z_2 + n'_3 B_3 + n'_3 Z_3 + C'_3\right) \quad (16)$$

included among these.

$$\begin{aligned} n'_1 &= n_1 - n_3 (1-p), \\ n'_2 &= n_2 - n_3 (1-p), \\ n'_3 &= \min\{ (1 - X_1 p'_1) n'_1, (1 - X_2 p'_2) n'_2 \} \\ p'_1 &= \frac{n_1 p_1}{n'_1} \\ p'_2 &= \frac{n_2 p_2}{n'_2} \\ p' &= 1 - (1-(1-X_1)p'_1)(1-(1-X_2)p'_2)(1-p_3)\ldots \end{aligned} \quad (17)$$

With a large number of iterations, a higher accuracy of C4 can eventually be derived.

X4 = 1, that is, the case of dismantling the detected substandard products, the dismantling process must be after the inspection process, so we can see that, at this time, X3= 0 must be established. That is, when X3= 1 and X4 = 1 The expectation of the objective function is as follows:

$$\text{max profit } = \min\{ (1-p_1)n_1, (1-p_2)n_2 \} \times \text{price} - \sum_{i=1}^{4} C_i \quad (18)$$

Assuming that the number of parts 1 and 2 is 1000, find the maximum value of the objective function by enumeration in matlab and give the corresponding decision scheme.

Table 1. Profits Generated by Different Decision - making Schemes under Various Cases

Case	x1	x2	x3	x4	profit	Case	x1	x2	x3	x4	profit
1	0	0	0	0	11198	1	1	0	0	0	10398
1	0	1	0	0	9398	1	1	1	0	0	12420
1	0	0	1	0	9824	1	1	0	1	0	8724
1	0	1	1	0	7724	1	1	1	1	0	10260
1	0	0	0	1	11198	1	1	0	0	1	10398
1	0	1	0	1	9398	1	1	1	0	1	12420
1	0	0	1	1	-138310	1	1	0	1	1	-21297
1	0	1	1	1	-33672	1	1	1	1	1	-52191

From Table 1, it can be seen that for Case 1, x1= 1, x2= 1, x3= 0, x4= 0 is the optimal solution for Case 1, and the expected profit will reach the maximum value, which is 12420.

The optimal decision corresponding to each case is shown in Table 2:

Table 2. Maximum Profits and Their Optimal Decisions for Each Case

Case	x1	x2	x3	x4	profit
1	0	0	0	0	12420
2	0	0	0	0	11198
3	1	1	0	0	10260
4	1	1	1	0	5540
5	0	1	0	0	6968
6	0	0	0	0	18587

A plot of the relationship between each decision option and profit under different scenarios is shown in Figure 2:

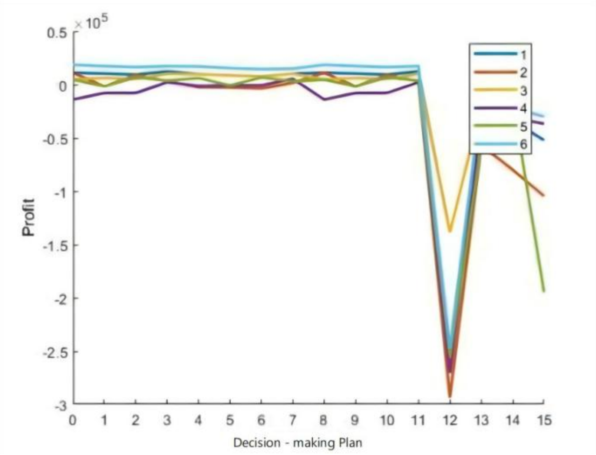

Figure 2. Distribution Map of Different Decisions and Profits in Each Case

As can be seen from the figure above, the decision scenarios with$x1 = 0, x2 = 0, x3 = 1$, and x4 = 1 have a negative impact on the enterprise, probably because during the continuous inspection and disassembly of the finished product, the qualified parts are assembled into the finished product, and the percentage of defective parts and components continues to increase, which further expands the cost of inspection.

4. Conclusions

This paper focuses on the decision-making problem in the production process of electronic products, and constructs a decision-making model based on the dynamic programming algorithm for the production process of spare parts assembly. Through the use of statistical hypothesis testing and efficacy analysis, we successfully design a sampling test scheme with as few tests as possible, specify the minimum sample size and the corresponding judgement conditions for testing spare parts under different confidence levels, and effectively reduce the testing cost. At the same time, we set up 0 - 1 decision variables, quantitatively analyse the decision-making in each stage of the production process, construct the cost and profit functions, and calculate the optimal decision-making and the maximum profit under each situation with the help of matlab enumeration.

The innovation of this research lies in the integrated use of multiple methods, taking into account the cost control of sampling inspection and multi-stage decision-making optimisation of the production process, which overcomes the shortcomings of traditional optimisation algorithms and machine learning methods in dealing with this kind of problem. The results of the study show that there are optimal decision-making solutions in different cases, such as case 1, the detection of spare parts 1 and 2, no detection of finished products and not disassembled substandard products when the profit can be up to 12,420, which provides a scientific basis for decision-making for enterprises.

However, this study may face challenges such as complex and changing production environments and increased data uncertainty in practical applications. In the future, we plan to further study how to better adapt the model to the complex and changing actual production scenarios, and explore the integration of more factors affecting production decisions, such as fluctuations in market demand and changes in raw material prices, in order to enhance the practicality and accuracy of the model and create greater economic benefits for enterprises

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