Quantitative Empirical Research on Portfolio Optimization Based on Sharpe Ratio and Market Index

Zijian Zhang *

Shanghai Academy of Fine Arts, Shanghai University, Shanghai, China

* Corresponding Author Email: rubyzzj123456789@shu.edu.cn

Abstract. With the increasing complexity of modern economic systems and the globalization of financial markets, investment has become a crucial means for individuals and institutions to achieve asset appreciation and hedge against inflation. However, a core dilemma emerges in investment practice: How should investors choose between scientifically model-driven rational decision-making and market experience-oriented passive strategies? This paper investigates two portfolio construction approaches: quantitative analysis based on mathematical models versus market cumulative return rates derived from historical market performance. Representative companies across various industries were selected, with historical stock market data collected for each entity. Employing quantitative empirical methods as the core framework, this study focuses on comparing the performance between Sharpe ratio-optimized investment portfolios and market benchmark returns, aiming to validate the effectiveness of rationally model-driven strategies in practical applications. Through comparative analysis of cumulative returns between Sharpe ratio-optimized portfolios and broad-based market indices, this research reveals that market benchmarks significantly outperform active strategies driven by mathematical models. The theoretical framework and empirical conclusions presented in this paper provide multidimensional support for investors seeking to establish scientific and rational investment decision-making systems.

Keywords: Sharpe ratio; market cumulative return rate; investment portfolio.

1. Introduction

The debate between active and passive investment strategies in real-world practice stems from diverging assumptions regarding market efficiency and investor rationality. On one hand, proponents of active investing argue that dynamic adjustments of cross-sector asset weights to capture asymmetric information, coupled with optimized risk diversification efficiency, can generate excess returns even in efficient markets. Conversely, advocates of passive strategies, grounded in the empirical reality of market efficiency and cumulative return rates, prioritize tracking broad market indices with minimal portfolio adjustments to achieve market-synchronous performance.

The relative merits of active versus passive strategies remain non-absolute, with no consensus emerging across academia and industry. Tong demonstrated the superiority of systematic strategies integrating predictive modeling, computational analysis, capital allocation, and risk control to achieve optimal returns [1]. Zhang derived coherent risk measures induced by active portfolio optimization [2]. Liu established the robustness of optimized portfolio performance across market cycles, modalities, and sample groups through benchmark asset allocation models [3]. Li highlighted active investing's efficacy in solving complex portfolio optimization problems under risk constraints, while Ni and Chen showed that active strategies enhance pricing models by exploiting market inefficiencies and dynamically adjusting return factors [4-6]. However, opposing studies suggest passive institutional ownership significantly reduces corporate financialization while promoting long-term investments [7]. Collectively, these studies reflect ongoing scholarly contention regarding portfolio strategy selection.

This study employs the classical Modern Portfolio Theory framework, with Sharpe ratio maximization as its methodological core, to empirically examine long-term performance disparities between mathematically driven active strategies and passive market-tracking approaches. Our findings reveal that market benchmarks consistently outperform model-driven active strategies. The central contribution lies in exposing a critical paradox: the mathematical rigor of theoretical models does not inherently translate to practical superiority. Although Sharpe ratio optimization theoretically defines an "optimal" portfolio, its real-world efficacy is constrained by parameter estimation errors, market efficiency levels, and dynamic complexities. These insights challenge the universal applicability of traditional asset allocation theories while providing empirical guidance for investors. Future research should explore dynamic coupling mechanisms between theoretical models and market realities to develop more resilient investment paradigms.

2. Data

The research data in this paper were sourced from Investing.com (https://cn.investing.com), covering the period from July 1, 2024, to November 19, 2024. The selected entities include: Coca-Cola Bottling Co Consolidated (NASDAQ: COKE), Nike Inc (NYSE: NKE), Tesla Inc (NASDAQ: TSLA), JPMorgan Chase & Co (NYSE: JPM), and United Airlines Holdings Inc (NASDAQ: UAL). These companies represent prominent leaders in their respective industries, ensuring the accuracy and standardization of the dataset.

Among the five corporations, the majority exhibited positive return rates, with an overall average return rate in positive territory. Tesla Inc demonstrated the highest expected return rate at 21.9%, alongside the lowest return rate of -12.33%. United Airlines Holdings Inc (UAL) achieved the highest mean expected return rate of 0.8% during the observation period (See Table 1 and Fig. 1).

Table 1. Descriptive statistics of these selected assets from 2024.7.1 to 2024.11.19

	Nike	Tesla	Coke	JPM	UAL
MAX	0.0684	0.2192	0.0833	0.1154	0.1244
MIN	-0.0677	-0.1233	0.0628	-0.0519	0.0700
MEAN	-0.0000	0.0068	0.0014	0.0020	0.0070
Variance	0.0003	0.0022	0.0003	0.0003	0.0008
Standard Deviation	0.0178	0.0470	0.0185	0.1842	0.0299

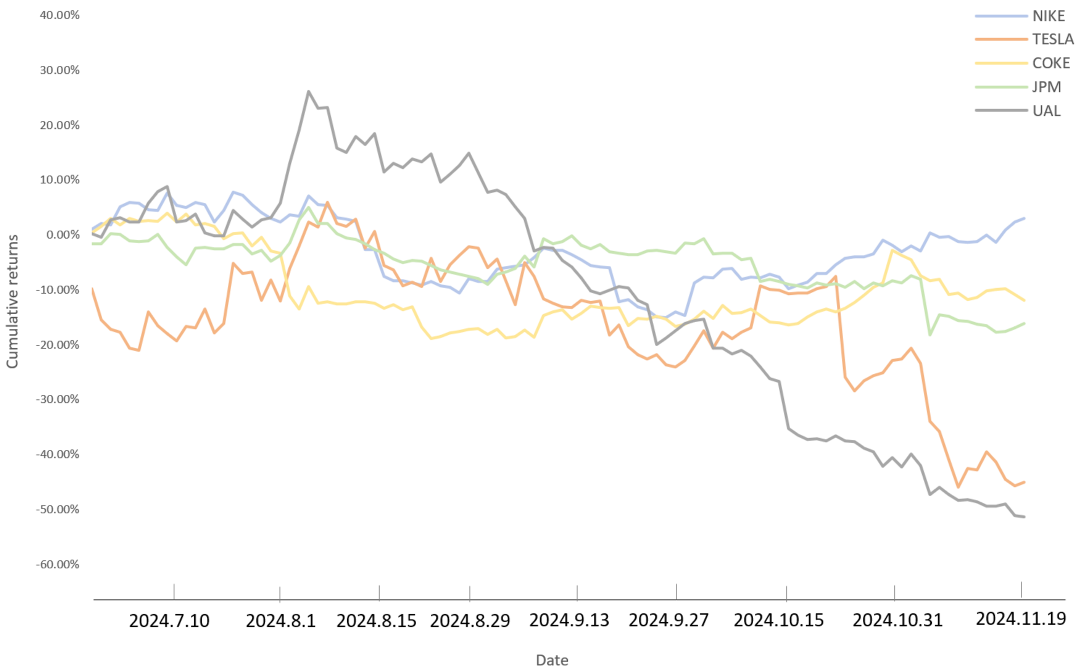

Fig. 1 Cumulative returns of the six stocks

Fig. 1 visually presents the cumulative return rates of the five selected companies. United Airlines Holdings Inc (UAL) exhibits both the highest and lowest cumulative returns, indicating the most significant volatility among the cohort.

3. Method

The Sharpe ratio serves as a cornerstone tool in portfolio research because it establishes a unified quantitative framework for portfolio optimization, performance evaluation, and capital allocation by standardizing the risk-return relationship [8]. The maximum Sharpe ratio model has become one of the most widely adopted approaches in portfolio theory precisely because it achieves an optimal equilibrium between returns and risks. Rather than indiscriminately pursuing high returns, the model prioritizes maximizing excess returns per unit of risk undertaken. By optimizing the Sharpe ratio, investors can identify portfolios with the most favorable risk-return trade-offs, thereby avoiding excessive risk exposure. The Sharpe ratio is mathematically expressed as follows.

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p} \quad (1)$$

In the equation above: R_p represents the expected return of the portfolio, R_f denotes the risk-free rate, and σ_p stands for the standard deviation of the portfolio's returns. The numerator of the equation represents the portfolio return minus the risk-free return, reflecting the additional reward for taking on risk. The denominator uses the standard deviation to measure the volatility of returns, where greater volatility indicates higher risk. In this formula, a higher Sharpe Ratio signifies superior excess return per unit of risk, indicating a more cost-effective portfolio. A negative value implies that the portfolio's performance is worse than the risk-free rate. When comparing different strategies, such evaluations should be conducted under the same time period and market conditions.

4. Result

Calculate the expected return using the opening and closing prices from the preceding 70 days (July 1, 2024, to October 8, 2024), and identify the risk-free rate. Assuming the sum of weights across all companies equals 1, define a set of random weights that do not yield negative returns.

After obtaining the random weights, combine them with the expected return of each company based on the average return and the variance-standard deviation table from the preceding 70 days to derive the maximum Sharpe Ratio. Once the maximum Sharpe Ratio is determined, use Monte Carlo simulation under the constraint that the sum of weights across all companies equals 1 to calculate the final weights for each company over the 70-day period (July 1, 2024, to October 8, 2024).

Monte Carlo simulation is a method that employs large-scale random number generation to model uncertainties within a system [9-10]. After defining the probability distributions of input variables, it uses computer-generated random numbers conforming to these distributions to simulate the behavior of uncertain systems. In the table, Tesla Inc. has the highest weight at 42.05%, while Nike and JPMorgan Chase (JPM) are assigned to zero weights. Tesla's annualized expected return is significantly higher than other assets. JPMorgan Chase exhibits relatively stable returns (mostly within ±2%) but lacks attractive excess returns. Nike shows significant declines on certain dates, with an insufficient magnitude of positive returns (See Table 2).

Table 2. Weight of Portfolio

Weight	
Nike	0.0000
Tesla	0.4205
Coke	0.0486
JPM	0.0000
UAL	0.5309
Expected Return	0.0035
Variance	0.0007
Standard Deviation	0.0266
Risk Free	0.0025
Sharpe Ratio	0.0392

After determining the investment weights for each company in the portfolio, this study validates the practical performance of the optimal portfolio by calculating its actual return from July 1, 2024, to November 19, 2024, under the given weights. The cumulative return for days 70-100 (October 9, 2024, to November 19, 2024) is computed and compared with the cumulative market return of the NASDAQ Composite Index. As a core global financial market, the NASDAQ index serves as an essential tool for investors to allocate capital across industries. If the optimal portfolio's cumulative return over the latter 30-day period outperforms the NASDAQ Composite Index, the portfolio is deemed effective (See Fig. 2).

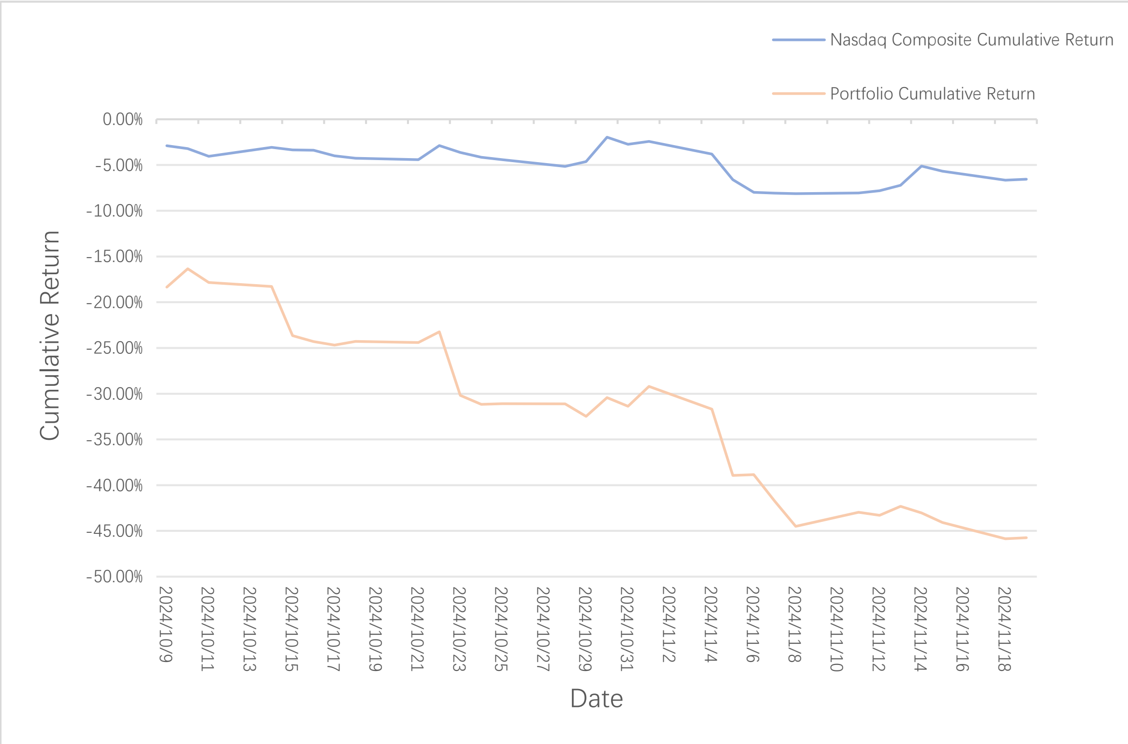

Fig. 2 Comparison between Nasdaq Composite and the constructed portfolio

The cumulative returns calculated using Nasdaq data for days 71 to 100 (October 9, 2024, to November 19, 2024) significantly outperformed the cumulative returns predicted by the Sharpe Ratio based on historical returns, with the former reaching a maximum value of -1.96% compared to the latter's -16.34%. This divergence stems from the fact that Nasdaq constituents are highly liquid large-cap stocks benefiting from real-time institutional capital inflows. For ordinary investors, directly holding Nasdaq index funds may represent a simpler and more efficient investment choice.

5. Conclusion

With the increasing complexity of modern economic systems and the deepening globalization of financial markets, investment activities have become a critical tool for individuals and institutions to achieve asset appreciation and hedge against inflationary pressures. Simultaneously, the trade-offs between rationality and intuition, as well as active versus passive strategies, in investment decision-making have grown increasingly prominent. Traditional financial theories advocate rational decision-making grounded in scientific models, yet in real-world market environments, passive strategies based on empirical observations and market behaviors remain widely adopted. The divergence in actual return performance between these approaches has long been a key focus of both academic and practical research.

Against this backdrop, this study employs quantitative empirical methods to systematically compare the performance of active investment strategies driven by scientific models with passive strategies represented by market benchmarks. The research design involves selecting representative publicly traded companies across industries, collecting historical market data, constructing an actively optimized portfolio based on the Sharpe Ratio, and conducting systematic comparative analysis against corresponding market benchmarks. Through backtesting and statistical validation, this paper further confirms the significant superiority of market benchmark strategies over model-driven active strategies in practical applications, revealing that active strategies fail to generate statistically significant excess returns.

However, this study has certain limitations. First, the optimization process primarily relies on the Sharpe Ratio for risk adjustment, without fully accounting for tail risks and asymmetric return characteristics. Future research could incorporate supplementary validation using other metrics. Second, the sample selection criteria and industry coverage may limit the generalizability and extrapolation of the findings. Lastly, the analysis is predominantly based on historical backtesting, which may not fully capture the potential impacts of macroeconomic cycles or investor behavior on strategy effectiveness.

Overall, this paper provides valuable empirical insights into understanding the performance differences between active and passive investment strategies, while also highlighting directions for methodological refinement and deeper exploration in subsequent studies.

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