Building And Testing a Minimum Variance Portfolio

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Abstract. This paper investigates portfolio risk management using historical market data from six major U.S. companies operating across key sectors such as consumer goods, technology, and financial services. This paper applies the minimum variance portfolio (MV) model to a dataset covering 100 daily returns between March and July 2003. The training sample, consisting of the first 70 observations, is used to estimate the covariance matrix and derive optimal portfolio weights. The minimum variance portfolio assigns a large weight to Procter & Gamble. However, it gives negative weight to some assets, including Southwest Airlines and Cisco Systems. The study uses the remaining 30 observations for out-of-sample testing. It compares the cumulative returns of the minimum variance portfolio with those of the S&P 500 index. Results indicate that the S&P 500 outperforms the minimum variance portfolio during the test period. The index achieves a cumulative return of +0.74%, while the portfolio posts a loss of -0.32%.

Keywords: Portfolio; Minimum variance; U.S.

1. Introduction

Constructing diversified investment portfolios is a core objective in finance. These portfolios aim to deliver stable returns or maximize investor gains. Effective portfolio design helps manage risks while targeting optimal performance. A scientifically constructed investment portfolios can maximize returns at a given risk level or minimize volatility at a target return. This dual focus has driven extensive academic and practical research [1].

Scholars have developed numerous models to refine portfolio strategies. Markowitz's Modern Portfolio Theory established a foundational framework. It proposed balancing risk and return through mean-variance optimization. Later studies expanded this work [2]. For example, Jegadeesh and Titman identified profitability in momentum-driven trading [3]. Fama and French introduced a three-factor model to better explain asset returns [4]. Despite progress, debates continue about active versus passive strategies. Some argue active portfolios rarely outperform passive investments like the S&P 500 [5].

Research findings remain divided. Fama and French supported the efficiency of broad market indices [6]. In contrast, Blitz and van Vliet showed minimum variance portfolios reduce risks during market turbulence [7]. Hsu et al. reported smart beta strategies combining low volatility and quality metrics generated 2.3% annual excess returns over the S&P 500 (2000–2017) [8]. However, Ang et al. noted such gains weaken when transaction costs and liquidity limits are considered [9]. These contradictions highlight how market conditions and investor constraints shape portfolio outcomes. By analyzing real-world data, this research provides insights into the practical challenges and limitations of minimum variance strategies, emphasizing the importance of empirical validation when applying portfolio theories to real investment decisions.

2. Data

This paper uses historical market data from six large U.S. companies. These companies operate in key sectors such as consumer goods, technology, and financial services. Their stock price movements reflect broader sector trends. The dataset contains core trading metrics, including daily opening prices, closing prices, and trading volumes. The data has been processed into a structured format for analytical purposes. Details of these six companies are given in Table 1 below.

Table 1. Details of the Eight companies

Ticker	Company (Sector)	Rationale for inclusion
WFC	Wells Fargo & Co. – Financials	Large commercial bank that captures the performance of traditional U.S. banking.
LUV	Southwest Airlines Co. – Industrials (Airlines)	Pioneer of the low-cost-carrier model; proxy for cyclical transport demand.
CSCO	Cisco Systems Inc. – Information Technology	Global leader in networking hardware and enterprise software.
PG	Procter & Gamble Co. – Consumer Staples	Staple household-goods maker, valued for defensive earnings.
KO	The Coca-Cola Company – Consumer Staples (Beverage)	Iconic beverage brand with stable cash flows.
MCD	McDonald's Corporation – Consumer Discretionary	World's largest quick-service restaurant chain; captures discretionary spending.

These firms are popular with both retail and institutional investors, enjoy deep trading liquidity, and together provide broad sector diversification that is essential for portfolio‑level risk analysis [10].

This paper collects the daily prices of the selected companies from 3 March 2003 to 18 July 2003. (101 trading sessions). Prices are converted to one‑day simple returns.

$$Rt = \frac{Pt}{P(t-1)} - 1 \tag{1}$$

To enable an out‑of‑sample performance check, the 100‑day return vector is split as follows. Training window: 3 Mar 2003 – 6 Jun 2003 (70 observations) – used to estimate covariances and derive portfolio weights. Test window: 9 Jun 2003 – 18 Jul 2003 (30 observations) - used to evaluate realised performance [11].

From Table 2, this paper observes that McDonald's (MCD) posts the highest average daily return (0.48 %), whereas Procter & Gamble (PG) exhibits the lowest return variance, underscoring its defensive nature. Southwest Airlines (LUV) enjoys the largest single-day gain (8.25 %), but Coca-Cola (KO) suffers the steepest one-day loss (-6.18 %), highlighting the dispersion of risk-reward profiles across sectors.

Table 2. Descriptive statistics of the selected assets (daily returns, 3 Mar – 18 Jul 2003)

	WFC	LUV	CSCO	PG	KO	MCD
Mean	0.0014	0.0037	0.0027	0.0011	0.0013	0.0048
Variance	0.0002	0.0006	0.0005	0.0001	0.0002	0.0006
Max Return	0.0362	0.0825	0.0575	0.0252	0.0549	0.0923
Min Return	–0.0320	–0.0582	–0.0415	–0.0183	–0.0618	–0.0666

3. Method

The minimum variance (MV) portfolio model focuses on producing the smallest possible total risk, regardless of return expectations. The advantage of the minimum variance model lies in its ability to minimize risk and maximize resource allocation efficiency through mathematical optimization and dynamic adaptability. This model is based on covariance matrix or precision matrix, which can find the configuration scheme with the minimum variance under given constraints, effectively reducing data dispersion or portfolio volatility. Its core value is reflected in its ability to dynamically adjust. In long-term practice, the minimum variance model can not only withstand the impact of extreme market volatility on investment portfolios but also improve system stability through real-time adjustment of strategies in industrial control, production management, and other fields, ultimately achieving the global optimal balance of risk and return. It is well-suited here because it captures diversification benefits, minimizes portfolio volatility, and has a closed-form solution, which limits estimation error and allows transparent replication with a relatively small training sample.

Let

Σ denote the N×N covariance matrix of asset returns,

1 denote the vector of ones.

The minimum variance portfolio weights www solve:

$\min_{w} w' \Sigma w$ subject to $w'1 = 1$

The analytic solution is as follows.

$$w = \frac{\Sigma^{-1}1}{1'\Sigma^{-1}1} \tag{2}$$

This solution distributes capital across assets to achieve the lowest possible portfolio variance, while fully investing the available capital (weights sum to one). No return targets are implemented; the focus is solely on risk minimization.

Minimum variance portfolios appeal to very cautious investors. They aim to lower the swings in value while maintaining some market exposure.

4. Result

This paper divides the 100 daily return observations into two groups. The first 70 observations (March 3 – June 6, 2003) form the training sample, and the remaining 30 observations (June 9 – July 18, 2003) form the test sample. Applying the minimum-variance formula to the 6 × 6 covariance matrix from the training window gives the weights reported in Table 3.

Table 3. Minimum-variance weights (training window)

Ticker	WFC	LUV	CSCO	PG	KO	MCD
Weight	0.226	–0.096	–0.043	0.882	–0.016	0.046

For each test-day, this paper multiplies the return vector rt by the weight vector www to get the portfolio return rpt. This paper compounds these daily profits through$\prod_{k=1}^{r}\left(1 + r_{p,k}\right) - 1$.

This paper repeats the same steps for the SPX series, so the two lines share one scale.

Figure 1 illustrates the cumulative return trajectories of the minimum variance portfolio and the S&P 500 (SPX) index over the 30-day test period (June 8 to July 15, 2003). The SPX index demonstrates a consistent upward trend, achieving a final cumulative return of +0.74%. In contrast, the minimum variance portfolio exhibits notable volatility, with initial gains followed by a sustained decline, ultimately yielding a cumulative loss of –0.32%. This divergence underscores the underperformance of the optimized portfolio relative to the passive benchmark during the evaluation window. Table 4 compares the total growth over the 30-day test window. Fig. 1 shows the full paths.

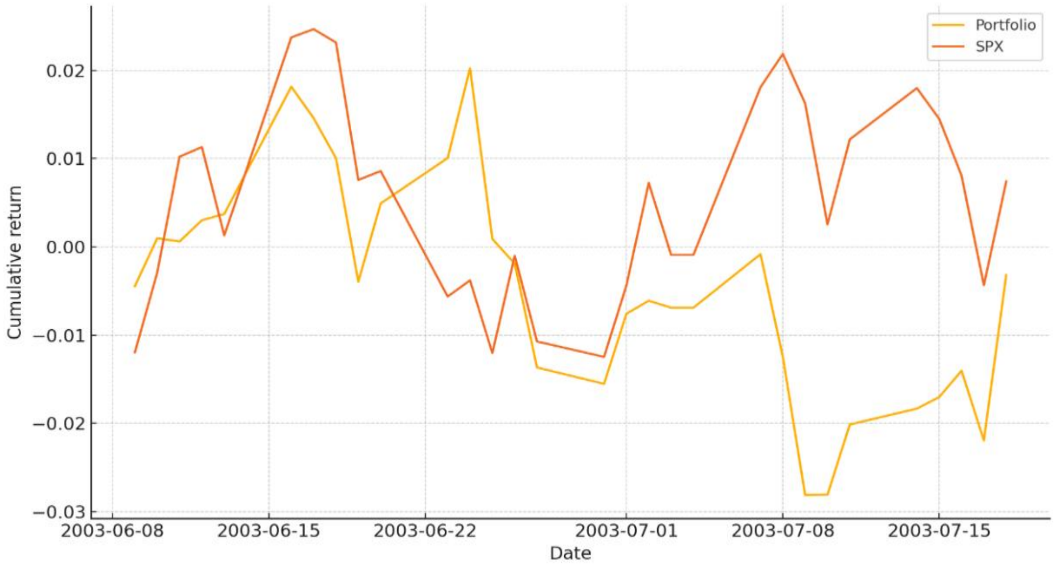

Fig. 1 Portfolio vs SPX (Days 71-100)

Table 4. Comparison of minimum variance portfolio and the market index

Strategy	Cumulative return
Minimum-variance portfolio	–0.32 %
SPX index	+0.74 %

During the 30-day test period, the S&P 500 index achieves a higher cumulative return than the minimum variance portfolio.

5. Conclusion

This paper explores a minimum variance portfolio strategy. It uses historical return data from six major U.S. companies. The training sample includes 70 daily observations. It is used to calculate the portfolio weights. The test sample includes 30 daily observations. It is used to check how the portfolio performs out of sample. The results show that the minimum variance portfolio did not perform as well as the S&P 500 index. The portfolio had a cumulative return of –0.32%, while the S&P 500 index achieved +0.74%. These results suggest that applying theoretical models in real markets can be challenging.

However, this paper has a few limitations. The sample period is short. It only covers a specific market period. This limits how much the findings can apply to other times. The model also allows short sales, which is not always practical for many investors.

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