Optimization Research on Crop Planting Strategies Based on Multi-objective Optimization Model and Game Theory

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Abstract. This paper aims to promote rural revitalization and support the long-term development of national agriculture by optimizing planting strategies, developing organic farming according to local conditions, and rationalizing the use of cultivated land. Based on the data related to cultivated land area in North China from 2017 to 2022, a multi-objective function optimization model is constructed, which comprehensively considers constraints such as land type, crop planting area limitations, crop continuous cropping effects, and legume crop rotation requirements. On this basis, the target-MOTAD model and importance sampling-Monte Carlo simulation are introduced to establish a comprehensive objective function for maximizing revenue and minimizing risk, which is solved by a hybrid genetic algorithm. Furthermore, complementarity coefficients and substitution coefficients are introduced to establish a market response model based on principal-agent game, which is dynamicized and solved by the NSGA-II algorithm, aiming to provide scientific decision support for agricultural producers and improve their revenue.

Keywords: Planting strategy optimization, Target-MOTAD model, Hybrid genetic algorithm, Principal-agent game, NSGA-II.

1. Introduction

By accurately identifying and implementing optimal planting schemes, we aim to maximize production efficiency, reduce planting risks, and enhance farming and field management. However, planning the best strategies faces challenges: First, the diverse types of rural farmland and greenhouses make model building and solving complex due to multiple factors. Second, the substitutability and complementarity between crops require special models for quantification. Third, market fluctuations affect crop sales, costs, yields, and prices, complicating planting decisions.

This study constructs a multi-objective optimization framework to formulate crop planting schemes. Drawing from Rong Bo's supply point location optimization via multi - objective integer linear programming[1], it shortens logistics time and integrates Aliano Filho's team's planting - harvesting period matching principle[2].The core objective function combines market sales and planting costs, referencing Abdulbaki's water resource allocation model[3]. For risk control, it uses Jha's target - MOTAD model[4] and Doorn's importance sampling - Monte Carlo simulation[5], minimizing and quantifying climate and market risks. Adewumi's farm risk planning study[6] and von Czettritz's climate - adaptive planting planning[7] underpin its theoretical validity. The algorithm combines genetic algorithm's global search and simulated annealing's local search, evaluating solutions with the Metropolis criterion, inspired by Abdolrasol's work on neural network parameter optimization[8]. It predicts 2024 - 2030 planting schemes, introduces complementarity and substitutability coefficients for crop interactions, and models market reactions via a Stackelberg game. The NSGA - II - based solution strategy draws on Yuan's team's ranking and crowding evaluation functions for workshop scheduling[9], and Haber's team's Cbenh - Olsen distance technology optimizes Pareto - front solutions[10]. Ultimately, it achieves collaborative crop - combination optimization.

2. Model introduction

2.1. Multi-objective optimization

Multi-objective optimization refers to the need to optimize multiple objectives simultaneously in a given scenario, where decision variables are the independent variables of the problem, objective functions are the indicators to be optimized, and constraint conditions are the limitations on decision variables.

2.2. Target-MOTAD Model

To control price and yield fluctuation risks of crops, the target-MOTAD model is introduced. This model minimizes deviations by calculating uncertain deviations of risks under natural states, achieving the goal of risk minimization. In the formula, cr,j represents the unit revenue of crop j under risk state r, and pr represents the occurrence probability of natural state r.

$$\mathrm{Min}\, V = \sum_{r=1}^{s} p_r ((R - C - \sum_{i=1}^{n} c_{r,i} x_i)^2) \tag{1}$$

2.3. Importance Sampling-Monte Carlo Model

To address uncertainties in expected crop sales volume, yield per mu, planting costs, and sales prices, this paper uses the importance sampling-Monte Carlo model for processing. In the sampling weight calculation formula, p(x) is the distribution of variables in real situations, and q(x) is the distribution used in simulations.

$$\omega(x) = \frac{p(x)}{q(x)} \tag{2}$$

2.4. Hybrid Genetic Algorithm

To solve complex multi-objective optimization problems, this paper combines genetic algorithms with simulated annealing algorithms, leveraging the global search capability of genetic algorithms and the local search capability of simulated annealing to find optimal planting strategy solutions.

(1) Randomly generate a population, where each individual corresponds to a planting scheme. Each individual Xk can be represented by a matrix, where rows represent crop j, columns represent plot i, quarters t and years a correspond to the time dimension, Xk represents the k-th individual in the population, and N is the population size.

$$X_k = \{x_{i,j,t,a}^k\}, \quad k = 1,2,...,N \tag{3}$$

(2) Calculate the fitness function to obtain the fitness value of each planting scheme Xk.

$$\begin{aligned} f_k = Revenue(X_k) = &\sum_{a=2024}^{2030} w_a \sum_{i=1}^{n} \sum_{j=1}^{m} (\min(x_{i,j,t}^k \times Y_{i,t}, D_{i,t}(X_k)) \times P_{i,t}(X_k) \times R_{i,j,t}) \\ &- \sum_{t=2024}^{2023} w_a \sum_{i=1}^{n} \sum_{j=1}^{m} C_{i,t} \times (1 - S_{i,t}) \times x_{i,j,t}^k \end{aligned} \tag{4}$$

(3) Conduct global search across the entire solution space to avoid local optima. First, use tournament selection for selection operations, then perform mutation operations by changing the planting area of a crop on a plot in a given year, and finally perform crossover operations.

$$\begin{cases} X_1^{'} = X_1[1:t], X_2[t+1:T] \\ X_2^{'} = X_2[1:t], X_1[t+1:T] \end{cases} \tag{5}$$

(4) Introduce simulated annealing for local search to rapidly optimize near screened better solutions, using the Metropolis criterion to determine whether to accept new solutions.

$$P = \exp\left(\frac{-(f_k - f_k^{'})}{T}\right) \tag{6}$$

2.5. NSGA-II Algorithm

Considering the potential substitutability and complementarity between various crops, which complicates objective functions and tightens constraint conditions, this paper uses the NSGA-II algorithm, introducing two key techniques: non-dominated sorting and crowding distance. The technical roadmap is shown in Figure 1.

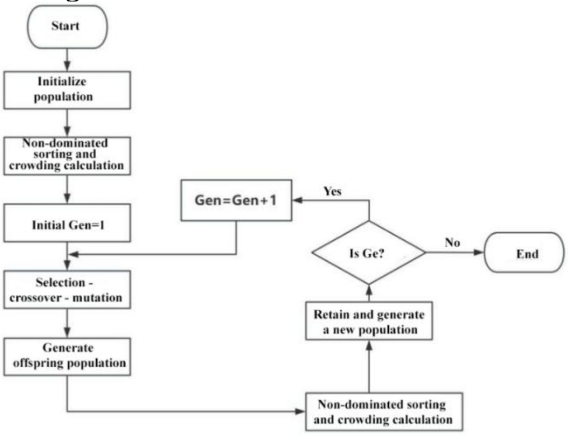

Figure 1. NSGA-II algorithm flowchart

3. Results

3.1. Descriptive Statistics

This paper first performs descriptive statistics and visualization on collected data. (The data comes from https://www.mcm.edu.cn/html_cn/node/a0c1fb5c31d43551f08cd8ad16870444.html) The sales volume of food crops such as wheat and vegetable crops such as Chinese cabbage both exceed 125,000 jin, ranking among the top in sales. In total profits, besides the highly profitable crops mentioned above like wheat, millet, and Chinese cabbage, edible fungi crops such as elm yellow mushrooms and morel mushrooms also yield significant profits.

Wheat has the largest overall planting area and is relatively dispersed, with 27 mu on hillside land, 80 mu on flat dry land, and 115 mu on terraced fields. Following wheat are millet, soybeans, corn, and mung beans, while the planting areas of cabbage, romaine lettuce, and leaf lettuce are relatively very small. Terraced fields and flat dry land have the largest areas, suitable for most crops, and can be considered the main plot types.

3.2. Establishment of Multi-Objective Optimization Model

3.2.1 Basic Model

(1)Assuming total production must exceed sales volume, the parameter variables are defined as follows in the table 1:

Table 1. Parameter Variable Definition Table

symbol	connotation
$x_{i,j,t}$	Planting area of crop j on plot i in quarter t
$x_{i,j,t,a}$	Planting area of crop j on plot i in quarter t of year a
$P_{j,t}(x)$	Price of crop j in quarter t, dependent on the village's planting strategy
$D_{j,t}(x)$	Demand for crop j in quarter t, adjusted according to the village's planting area
Y_j	Yield per mu of crop j
$C_{i,j}$	Unit cost of planting crop j on plot i
A_i	Area of plot i
$R_{i,j,t}$	Risk adjustment coefficient for planting crop j on plot i in quarter t
S_i	Government subsidy coefficient for crop i

(2)Different revenue functions are established for two scenarios: unsold excess production and reduced-price disposal of excess production. In the unsold scenario, crops exceeding market demand cannot be sold, and the market sales function is as follows:

$$\text{Max} R = \sum_{a=2024}^{2030} W_a \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{t=1}^{2} D_{j,t,a}(x) \times P_{j,t,a}(x) \times R_{i,j,t,a} \tag{7}$$

In the reduced-price sales scenario, it is assumed that crops exceeding market demand are sold at 50% of the market price, where W_a is a time-segmented adaptive weight that can be assigned different weights according to the importance of different years. The market sales function is as follows:

$$\begin{aligned} \text{Max} R = & \sum_{a=2024}^{2030} W_a \sum_{i=1}^{n} \sum_{j=1}^{m} D_{i,t,a}(x) \times P_{i,t,a}(x) \\ & + (x_{i,j,t,a} \times Y_{i,t,a} - D_{i,t,a}(x)) \times 0.5 \times P_{i,t,a}(x) \times R_{i,j,t,a}) \end{aligned} \tag{8}$$

While ensuring maximum market sales, it is necessary to minimize planting costs, with the cost function as follows:

$$\text{Min}\, C = \sum_{a=2024}^{2030} W_a \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{t=1}^{2} C_{i,t,a} \times (1 - S_{i,t,a}) \times x_{i,j,t,a} \tag{9}$$

The total objective function is as follows:

$$\text{Max} Z = R - C \tag{10}$$

(3)Establishment of constraint conditions

Flat dry land, terraced fields, hillside land: Only one season of food crops can be planted, and no second season can be planted:

$$\begin{cases} \sum_{j \in S_{grain}} x_{i,j,1} > 0 & \text{Flat dry land, terraced fields, hillside land} \\ \sum_{j=1}^{m} x_{i,j,2} = 0 & \text{Flat dry land, terraced fields, hillside land} \end{cases} \tag{11}$$

Irrigated land: Only one season of rice or two seasons of vegetables can be planted, and the second season can only plant specific vegetables:

$$\begin{cases} \sum_{j \in S_{paddy}} x_{i,j,1} + \sum_{j \in S_{veg1}} x_{i,j,1} > 0 & \text{Irrigated land} \\ \sum_{j \in S_{veg2}} x_{i,j,2} \le 1 & \text{Irrigated land} \end{cases} \tag{12}$$

Ordinary greenhouses: Must plant one season of vegetables and one season of edible fungi:

$$\begin{cases} \sum_{j \in S_{veg}} x_{i,j,1} > 0 & \text{Ordinary greenhouses} \\ \sum_{j \in S_{fungus}} x_{i,j,2} > 0 & \text{Ordinary greenhouses} \end{cases} \tag{13}$$

Smart greenhouses: Must plant two seasons of vegetables each year:

$$\sum_{j \in S_{veg}} x_{i,j,1\&2} > 0 \quad \text{Smart greenhouses} \tag{14}$$

Non-negativity and binary variable constraints: Decision variable $x_{i,j,t}$ is a binary variable indicating whether to plant the crop:

$$x_{i,j,t} \in \{0,1\} \quad \forall i, \forall j, \forall t \tag{15}$$

Continuous cropping constraint: Need to avoid the impact of continuous cropping of the same crop on the same plot for multiple years:

$$x_{i,j,t} \times x_{i,j,t+1} = 0, \quad \forall i, j, t \tag{16}$$

Legume crop rotation constraint: According to rotation requirements, each plot must plant legume crops at least once every three years to promote the growth of other crops through the nitrogen-fixing properties of legume root bacteria. Let legume crops be S_{bean}:

$$\sum_{j \in S_{bean}} (x_{i,j,1} + x_{i,j,2}) \geq 1 \quad \forall i \tag{17}$$

Crop non-dispersion constraint: The total planting area of each crop j cannot be less than the minimum threshold $A_{min,j}$, and there are plot area limitations, where the planting area of each plot cannot exceed the total area of the plot:

$$\begin{cases} \sum_{i=1}^{n} \sum_{t=1}^{2} A_i \times x_{i,j,t} \geq A_{min,j} & \forall j \\ \sum_{i=1}^{n} \sum_{t=1}^{2} A_i \times x_{i,j,t} \leq A_{max,j} & \forall j \end{cases} \tag{18}$$

3.2.2 Risk Minimization Model

On the basis of the above, to further minimize risks, the target-MOTAD model is introduced. Since the risk function is not directly proportional to other objectives, weights 1, 2, and 3 are assigned to the three objectives, with the target revenue function as follows:

$$\text{Max} Z = \omega_1 R - \omega_2 C - \omega_3 V \tag{19}$$

New constraint conditions are as follows:

Sales volume fluctuations: Sales of wheat and corn are expected to grow by 5%-10% annually, while sales of other crops fluctuate by 5%:

$$\begin{cases} S_{wheat,a} = S_{wheat,2023} \times (1 + r_{wheat,a}), & r_{wheat,a} \in (0.05, 0.10) \\ S_{corn,a} = S_{corn,2023} \times (1 + r_{corn,a}), & r_{corn,a} \in (0.05, 0.10) \\ S_{i,a} = S_{i,2023} \times (1 + r_{j,a}), & r_{i,a} \in (-0.05, 0.05) \end{cases} \tag{20}$$

Yield per mu fluctuations: Crop yields per mu are affected by factors such as climate, with fluctuations of ±10%:

$$Y_{j,t} = Y_{j,2023} \times (1 + \varepsilon_{i,t}), \quad \varepsilon_{i,t} \in (-0.10, 0.10) \tag{21}$$

Planting cost fluctuations: Planting costs increase by an average of 5% annually:

$$C_{i,t} = C_{i,2023} \times (1 + 0.05)^{t-2023} \tag{22}$$

Crop price fluctuations: Prices vary by crop type, with stable prices for grain crops, a 5% annual increase for vegetable crops, and a 1%-5% annual decrease for edible fungi crops, as specified by the following formula:

$$\begin{cases} P_{grain,t} = P_{grain,2023} \\ P_{veg,t} = P_{veg,2023} \times (1+0.05)^{t-2023} \\ P_{fungus,t} = P_{fungus,2023} \times (1-\varepsilon_{P,t}), \qquad \varepsilon_{P,t} \in (0.01, 0.05) \end{cases} \tag{23}$$

3.2.3 Substitutability and Complementarity Model

In real life, considering the potential substitutability and complementarity between various crops, this paper draws a relationship network diagram after consulting a large amount of relevant literature, as follows in the figure 2:

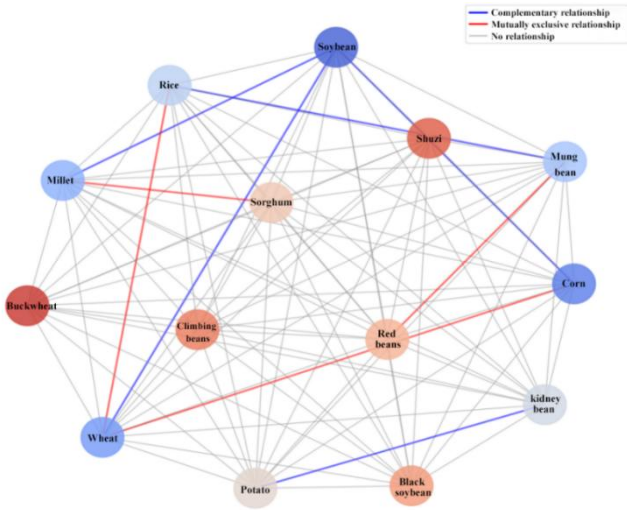

Figure 2. Network diagram of different crop relationships

Dynamic substitution-complementarity coefficients and dynamic risk adjustment coefficients are introduced to address the impacts of substitutability and complementarity between crops and the constraints of time evolution. The complementarity coefficient $c_{j,k,t,a}$ represents the additional revenue generated by collaboration with other crops, and the substitution coefficient $s_{j,k,t,a}$ represents the revenue loss caused by competition between crops. The profit function for crop j in quarter t of year a can be expressed as:

$$R'(x) = p_{j,t}\sum_{i=1}^{n} x_{i,j,t,a}Y_{i,j,t,a} + \sum_{k=1}^{n} c_{j,k,t,a}x_{k,i,t} - \sum_{k=1}^{n} s_{j,k,t,a}x_{i,j,t,a}x_{k,i,t,a} \tag{24}$$

The optimal strategy for each crop can be obtained by solving the Nash equilibrium through a dynamic game model.

$$x_{i,j,t,a} = argmax_{x_{i,j,t,a}} U(x) \mid x_{k,i,t,a} \text{ for } k \neq j \tag{25}$$

Considering risk aversion to fluctuations, a new risk objective function is established, where λ represents the degree of risk aversion.

$$\text{Min}\, V = \sum_{a=2023}^{2030} \lambda V(x) \tag{26}$$

$$\text{Max} Z = \omega_1 R' - \omega_2 C - \omega_3 V' \tag{27}$$

New constraint conditions are as follows:

$$x_{i,j,t,a} + s_{j,k,t,a}x_{k,i,t,a} \leq D_{j,t,a} \qquad \forall i,k,j,t,a \tag{28}$$

3.3. Calculation Results

Solving the basic multi-objective programming problem yields specific planting strategies. Agricultural producers prefer to plant legume and food crops, with less planting of edible fungi crops. In the second season, most cultivated land is used to plant vegetable crops, which is related to the climatic conditions in North China. Calculated profits show that in the first scenario, annual total profits are highest in 2028, reaching 6,797,152.59 yuan, and lowest in 2026, at only 5,646,912.18 yuan. A visual comparison of the 2024 annual total profits under the two scenarios is shown below in the figure 3:

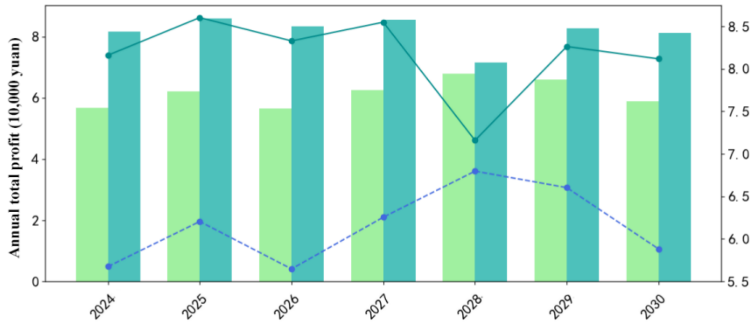

Figure 3. Comparison of Annual Total Profit between Two Scenarios

In the above figure, the solid line at the top represents total profits under the second scenario, and the dashed line at the bottom represents total profits under the first scenario. As shown, for crops exceeding expected sales volume, the solid line (second scenario) consistently outperforms the dashed line (first scenario), indicating that selling excess crops at a 50% discount is more profitable than letting them go to waste.

Considering potential planting risks, after considering the substitutability and complementarity between crops, another planting strategy is derived.There is a strong complementary relationship among wheat, corn, soybeans, mung beans, and millet, resulting in significant differences in planting areas between mung beans and soybeans, as well as between wheat, corn, and millet. Additionally, agricultural producers should balance edible fungi crops to increase overall profits.

3.4. Model Validation

This paper conducts Gaussian perturbation and robustness tests to analyze their impacts on planting strategies and revenues, ensuring the model's robustness and adaptability to minor fluctuations. The substitution coefficient fluctuation range is ±5%, with a Gaussian perturbation of standard deviation 2%; the complementarity coefficient fluctuation range is also ±5%, with a standard deviation of 2%; the sales-price correlation is set to ±5% fluctuation, with a standard deviation of 1%; and the cost-price correlation is also ±5%, with a standard deviation of 1%.

As shown in the figure4, the model maintains optimization effects robustly across different parameter combinations, indicating good sensitivity to multi-parameter adjustments and strong robustness.

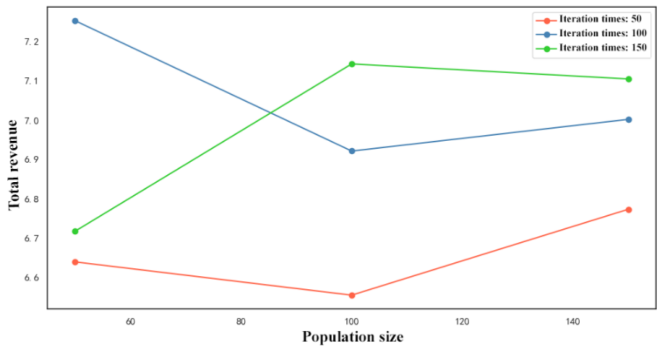

Figure 4. Sensitivity analysis of population size and iteration times to total incom

4. Conclusions

When sales volume, planting costs, yield per mu, and prices are stable but production may exceed sales, two strategies for dealing with overproduction - waste and 50% discount sales - were analyzed. Results show that from 2024 to 2030, the discount sales strategy significantly outperforms the unsold waste strategy. This suggests that reasonable price reductions can enhance agricultural revenue and reduce resource waste and losses due to overproduction.

Given market fluctuations in these factors, risk and uncertainty management analyses suggest that increasing the planting area of food crops and legumes while reducing edible fungi crops can help mitigate climate and market volatility risks.

Further adjustments based on crop complementarities reveal that under high risk tolerance, a planting scheme considering crop correlations may have slightly lower profits but stronger market adaptability.

In future research, it is advisable to integrate technologies like IoT and big data to collect real-time soil moisture and weather data, enhancing the model's responsiveness to dynamic environments. Expanding the research scope to include downstream links such as agricultural product processing and sales channels could help build an "planting-processing-marketing" optimization system. Additionally, combining carbon trading mechanisms with planting strategies could support agricultural low-carbon transformation while ensuring farmers' profits. These expansions aim to provide more comprehensive decision support for smart agriculture and rural revitalization, promoting a more efficient and sustainable agricultural industry.

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